# Class 23 in-class problems, 18.05, Spring 2022

## Concept questions

### Concept question 1. Overnight polling

During the presidential election season, pollsters often do 'overnight polls' and report a 'margin of error' of about $\pm 4\%$.

The number of people polled is in which of the following ranges?

(a) $0-\ 50$

(b) $50-100$

(c) $100-500$

(d) $300-600$

(e) $600-1000$

## Board questions

### Problem 1. Confidence intervals for a binomial

For a poll to find the proportion $\theta$ of people supporting X we know that a $(1-\alpha)$ confidence interval for $\theta$ is given by

$$\left[\bar{x}-\frac{z_{\alpha/2}}{2\sqrt{n}},\ \bar{x}+\frac{z_{\alpha/2}}{2\sqrt{n}}\right].$$

**(a)** How many people would you have to poll to have a margin of error of 0.01 with 95% confidence? (You can do this in your head.)

**(b)** How many people would you have to poll to have a margin of error of 0.01 with 80% confidence. (You'll want R or other calculator here.)

**(c)** If $n = 900$, compute the 95% and 80% confidence intervals for $\theta$.

### Problem 2. Pivoting: confidence intervals and non-rejection regions

This question gets at the relationship between confidence intervals and non-rejection regions.

**Main point:** For a sample with sample mean $\overline{x}$, the confidence interval consists of all values $\mu$ for which a NHST with null hypothesis mean $= \mu$ would not reject on seeing $\overline{x}$.

Assume we have independent data $x_1, \dots, x_n \sim N(\mu, \sigma^2)$, where $\mu$ is unknown and $\sigma$ is known.

**(a)** For null hypothesis $\mu = \mu_0$ give the two-sided non-rejection region for significance level $\alpha$.

**(b)** Call the data average $\overline{x}$. Give the $1-\alpha$ confidence interval for $\mu$.

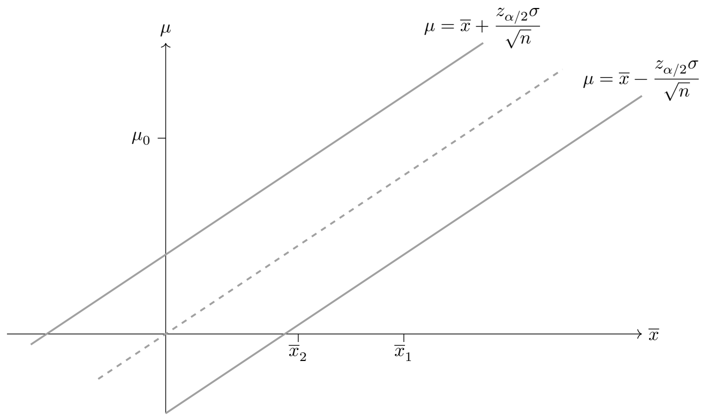
**(c)** Use the $\overline{x}, \mu$-plane below. Note the conveniently included guides.

(i) Plot the horizontal line segment at height $\mu_0$ showing the non-rejection region for $H_0 : \mu = \mu_0$ (significance level $= \alpha$).

(ii) Plot the horizontal line segment at other heights showing the non-rejection region for the corresponding $\mu$.

(iii) Plot the vertical line segments showing the $1-\alpha$ confidence intervals around $\overline{x}_1$ and $\overline{x}_2$

(iv) Plot the vertical line segment at other values of $\overline{x}$ showing the corresponding confidence interval.




Understand how the main point connects with your graph.

**Problem 3. Exact binomial confidence interval**

*This was not used in class, but it is a nice problem, so we included it here.*

Use this table of binomial(8,$\theta$) probabilities to:

**1.** find the (two-sided) rejection region with significance level 0.10 for each value of $\theta$.

**2.** Given $x = 7$, find the 90% confidence interval for $\theta$.

**3.** Repeat for $x = 4$.

| $\theta\backslash x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.430 | 0.383 | 0.149 | 0.033 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.3 | 0.058 | 0.198 | 0.296 | 0.254 | 0.136 | 0.047 | 0.010 | 0.001 | 0.000 |
| 0.5 | 0.004 | 0.031 | 0.109 | 0.219 | 0.273 | 0.219 | 0.109 | 0.031 | 0.004 |
| 0.7 | 0.000 | 0.001 | 0.010 | 0.047 | 0.136 | 0.254 | 0.296 | 0.198 | 0.058 |
| 0.9 | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 | 0.033 | 0.149 | 0.383 | 0.430 |

**Problem 4. Pivoting: Chi square confidence intervals for variance**

*This was not used in class, but it is a nice problem, so we included it here.*

Assume we have independent data $x_1, \ldots, x_n \sim N(\mu, \sigma^2_{\text{true}})$, where $\sigma_{\text{true}}$ is unknown and our parameter of interest.

Let $s^2$ be the sample variance. We know that $\dfrac{(n-1)s^2}{\sigma^2_{\text{true}}} \sim \chi^2(n-1)$. Thus,

$$P(c_{1-\alpha/2} \leq \frac{(n-1)s^2}{\sigma^2} \leq c_{\alpha/2} \,|, \sigma_{\text{true}} = \sigma) = 1 - \alpha.$$

Here, $c_{\alpha/2}$ is the right critical point for the $\chi^2(n-1)$ distribution.

Using this, for a two-sided significance test with with $H_0 : \sigma_{\text{true}} = \sigma$, the non-rejection region for $s^2$ at significance level $\alpha$ is

$$\frac{c_{1-\alpha/2}\,\sigma^2}{n-1} \leq s^2 \leq \frac{c_{\alpha/2}\,\sigma^2}{n-1}$$

Pivoting, we get the $1-\alpha$ confidence interval for $\sigma^2$ produced by the data is

$$\frac{(n-1)s^2}{c_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{c_{1-\alpha/2}}$$

Display this graphically on the $\sigma^2$-$s^2$ axes shown.