Name ______________________

18.05 Exam 1

6 problems. Do all your work on the paper provided

No books or calculators.

You may have one side of a 8.5×11 piece of paper with any information you like on it.

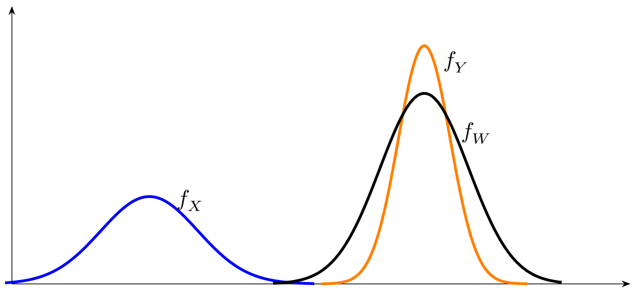
Simplifying expressions: Unless explicitly asked to do so, you don't need to simplify complicated expressions. For example, you can leave $\frac{1}{4} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{5}$ exactly as is. Likewise for expressions like $\frac{20!}{18!2!}$.

Table of normal probabilities: Use the table of standard normal probabilities at the end of this exam to do your computations for normal variables.

Cheat Sheet

Problem 0. (5 pts) **Be sure to attach your cheat sheet to your test.**

Problem 1. (10 pts: 4,6) **Concept/Quick questions**
(a) (No explanations are necessary.)
The plot shows the pdf for three independent random variables X, Y, W. All use the same horizontal and vertical scale.



Which random variable has the greatest variance?

(b) Suppose A and B are two events and $P(A) = 0.7$, $P(B) = 0.3$ and $P(A \cap B) = 0.25$. Compute each of the following

(i) Compute $P(A \cup B)$

(ii) Compute $P(A|B)$.

Problem 2. (15 pts: 10,5)
You create passwords as a string of 10 characters such that:

- 5 of the characters are letters (upper and lower case, i.e. 52 characters) with repetitions allowed,
- 3 are numbers { 0,1,2,3,4,5,6,7,8,9 } with repetitions allowed, and
- 2 are **distinct** symbols from the list of 5 symbols: { !, @, #, $, & }.

(a) How many passwords are there? (No need to simplify your answer.)

(b) With all locations for symbols, letters, or numbers in your 10 character password being equally likely, what is the probability that the two symbols are next to each other?

Problem 3. (25 pts: 10,5,5,5)
You have 5 four-sided and 3 six-sided dice. You put them in a cup, choose one at random, roll the die, and report the result.

Let D be the number of sides on the chosen die and let R be the result of the roll.

(a) Make a joint probability table for D and R. Be sure to include the marginal probabilities for D and R.

(b) What is the probability of rolling a 3?

(c) Compute $P(D = 4|R = 3)$.

(d) Are D and R independent?

Problem 4. (10 pts)
A quick screening test for a certain disease has three outcomes: positive, negative and uncertain. Suppose it has the following percentages.

For someone with the disease: positive 70%, negative 10%, uncertain 20%.
For someone without the disease: positive 10%, negative 60%, uncertain 30%.

Suppose also, that the prevalence of the disease in the population is 2%.

What is the probability that a random person who tests positive has the disease?

Problem 5. (25 pts: 5,5,5,5,5)
Two students, Xeno and Yolanda are meeting up for lunch. They plan on a time to meet at noon. Both have class before so neither will be early. Both have class that starts at 1pm, so they will both arrive between 0 and 1 hour late. Let X be the time in hours that Xeno arrives late and let Y be the time in hours that Yolanda arrives late.

Assume that the joint pdf of these random variables is $f(x, y) = 5/4 - xy$.

(a) Find the two marginal pdfs.

(b) Are X and Y independent?

(c) Find $E[X]$, $\text{Var}(X)$. **(For these, you need to simplify the fractions.)**

(Problem 5 continued on next page)

(Problem 5 continued)

(d) Compute the covariance $\mathrm{Cov}(X, Y)$ and correlation $\mathrm{Cor}(X, Y)$.

Hint: By symmetry you know the mean and variance of Y are the same as those for X.

For this part, there is no need to simplify fractions.

(e) Set up, **but do not compute** an expression computing the probability that Xeno and Yolanda arrive within 6 minutes (0.1 hours) of each other and that Yolanda arrives after Xeno.

Your integral will be over a region R in the unit square. You can leave your integral in the form $\iint_R h(x, y)\, dx\, dy$ and show R in a figure elsewhere on the page. The function $h(x, y)$ must be specified completely.

Problem 6. (10 pts)
A company manufactures solar panels. When homeowners install the panels, the state pays 50% of the cost. Because this subsidy is about to expire, the company wants to manufacture as many panels as it can in the next 20 days.

For a variety of reasons the number of panels it can manufacture in a day is a random variable with each day independent of the others. Suppose the daily output follows a so-called gamma distribution. The pdf of this distribution is not that complicated ($f(x) = \dfrac{x^4}{4! \cdot 10^{10}} \mathrm{e}^{-x/100}$), but we'll let Wikipedia tell us the mean and variance: mean = 500, variance = $5 \cdot 10^4$.

Estimate the probability that they will be able to manufacture more than 10,500 panels in the next 20 days.

Standard normal table of left tail probabilities.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-4.00	0.0000	-2.00	0.0228	0.00	0.5000	2.00	0.9772
-3.95	0.0000	-1.95	0.0256	0.05	0.5199	2.05	0.9798
-3.90	0.0000	-1.90	0.0287	0.10	0.5398	2.10	0.9821
-3.85	0.0001	-1.85	0.0322	0.15	0.5596	2.15	0.9842
-3.80	0.0001	-1.80	0.0359	0.20	0.5793	2.20	0.9861
-3.75	0.0001	-1.75	0.0401	0.25	0.5987	2.25	0.9878
-3.70	0.0001	-1.70	0.0446	0.30	0.6179	2.30	0.9893
-3.65	0.0001	-1.65	0.0495	0.35	0.6368	2.35	0.9906
-3.60	0.0002	-1.60	0.0548	0.40	0.6554	2.40	0.9918
-3.55	0.0002	-1.55	0.0606	0.45	0.6736	2.45	0.9929
-3.50	0.0002	-1.50	0.0668	0.50	0.6915	2.50	0.9938
-3.45	0.0003	-1.45	0.0735	0.55	0.7088	2.55	0.9946
-3.40	0.0003	-1.40	0.0808	0.60	0.7257	2.60	0.9953
-3.35	0.0004	-1.35	0.0885	0.65	0.7422	2.65	0.9960
-3.30	0.0005	-1.30	0.0968	0.70	0.7580	2.70	0.9965
-3.25	0.0006	-1.25	0.1056	0.75	0.7734	2.75	0.9970
-3.20	0.0007	-1.20	0.1151	0.80	0.7881	2.80	0.9974
-3.15	0.0008	-1.15	0.1251	0.85	0.8023	2.85	0.9978
-3.10	0.0010	-1.10	0.1357	0.90	0.8159	2.90	0.9981
-3.05	0.0011	-1.05	0.1469	0.95	0.8289	2.95	0.9984
-3.00	0.0013	-1.00	0.1587	1.00	0.8413	3.00	0.9987
-2.95	0.0016	-0.95	0.1711	1.05	0.8531	3.05	0.9989
-2.90	0.0019	-0.90	0.1841	1.10	0.8643	3.10	0.9990
-2.85	0.0022	-0.85	0.1977	1.15	0.8749	3.15	0.9992
-2.80	0.0026	-0.80	0.2119	1.20	0.8849	3.20	0.9993
-2.75	0.0030	-0.75	0.2266	1.25	0.8944	3.25	0.9994
-2.70	0.0035	-0.70	0.2420	1.30	0.9032	3.30	0.9995
-2.65	0.0040	-0.65	0.2578	1.35	0.9115	3.35	0.9996
-2.60	0.0047	-0.60	0.2743	1.40	0.9192	3.40	0.9997
-2.55	0.0054	-0.55	0.2912	1.45	0.9265	3.45	0.9997
-2.50	0.0062	-0.50	0.3085	1.50	0.9332	3.50	0.9998
-2.45	0.0071	-0.45	0.3264	1.55	0.9394	3.55	0.9998
-2.40	0.0082	-0.40	0.3446	1.60	0.9452	3.60	0.9998
-2.35	0.0094	-0.35	0.3632	1.65	0.9505	3.65	0.9999
-2.30	0.0107	-0.30	0.3821	1.70	0.9554	3.70	0.9999
-2.25	0.0122	-0.25	0.4013	1.75	0.9599	3.75	0.9999
-2.20	0.0139	-0.20	0.4207	1.80	0.9641	3.80	0.9999
-2.15	0.0158	-0.15	0.4404	1.85	0.9678	3.85	0.9999
-2.10	0.0179	-0.10	0.4602	1.90	0.9713	3.90	1.0000
-2.05	0.0202	-0.05	0.4801	1.95	0.9744	3.95	1.0000

$\Phi(z) = P(Z \leq z)$ for $N(0,1)$.

(Use interpolation to estimate z values to a 3rd decimal place.)

Extra paper