18.05 Problem Set 1, Spring 2022

Problem 1. (20 pts.) **(6 card draw)**
You can easily look up the probability of 5 card poker hands, e.g. https://en.wikipedia.org/wiki/Poker_probability

For this problem, let's consider hands with 6 cards. Here are two types:

Two-pair: Two cards have one rank, two cards have another rank, and the remaining two cards have two different ranks. e.g. $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 5\clubsuit, Q\diamondsuit, K\diamondsuit\}$

Three-of-a-kind: Three cards have one rank and the remaining three cards have three other ranks. e.g. $\{2\heartsuit, 2\spadesuit, 2\clubsuit, 5\clubsuit, 9\spadesuit, K\heartsuit\}$

Calculate the probability of each type of hand. Which is more probable?

Problem 2. (20: 10,10 pts.) **(Non-transitive dice)**
In class we worked with non-transitive dice:

Blue: 3 3 3 3 3 6; Orange: 1 4 4 4 4 4; White: 2 2 2 5 5 5.

(a) Find the probability that white beats orange, the probability that orange beats blue and the probability that blue beats white.

Can you line the dice up in order from best to worst? (Hint: this is why these are called 'non-transitive'.)

(b) Suppose you roll two white dice against two blue dice. What is the probability that the sum of the white dice is greater than the sum of the blue dice?

For hints on this problem and ways to make money with your dice, watch at least the first six minutes of the following video.

https://www.youtube.com/watch?v=zWUrwhaqq_c

Here a tree is used to organize the calculation rather than a table. We will discuss this method in Week 2.

Problem 3. (55: 5,5,10,5,10,10,10 pts.) **(Birthdays: counting and simulation)**
Ignoring leap days, the days of the year can be numbered 1 to 365. Assume that birthdays are equally likely to fall on any day of the year. Consider a group of n people, of which you are not a member. An element of the sample space S will be a sequence of n birthdays (one for each person).

(a) Define the probability function P for S. (This will depend on n.)

(b) Consider the following events:
A: "someone in the group shares *your* birthday"
B: "some two people in the group share a birthday"
C: "some three people in the group share a birthday"

Carefully describe the subset of S that corresponds to each event.

(c) Find an exact formula for $P(A)$. What is the smallest n such that $P(A) > 0.5$?

(d) Justify why n in part (c) is greater than $\frac{365}{2}$ without doing any computation. (We are looking for a short answer giving a heuristic sense of why this is so.)

(e) Use R simulation to estimate the smallest n for which $P(B) > 0.9$. For these simulations, let the number of trials be 10000. (You can reuse your code from Studio 1.)

For this value of n, repeat the simulation a few times to verify that it always gives similar results.

Using 10000 trials you saw very little variation in the estimate of $P(B)$. Try this again using 30 trials and verify that the estimated probabilities are much more variable. On your pset, give the results 7 runs of 30 trials using the value of n you just found.

(f) Find an exact formula for $P(B)$.

(g) Use R simulation to estimate the smallest n for which $P(C) > 0.5$. Again use 10000 trials. You will find that two adjacent values of n are equally plausible based on simulations. You may pick either one for your answer.

Note that it is much harder to find an exact formula for $P(C)$, so simulation is especially handy.