

PROFESSOR:

Hi, and welcome. Today we're going to do a problem about least squares approximations. Here's the question. Find the quadratic equation through the origin that is a best fit for these three points: (1, 1), (2, 5), and (-1, -2). I'll give you a minute to work that out on your own. You can hit the pause button now, and we'll be back in a minute to work the problem out together.

OK and we're back. So what's the first step in a problem like this? The first step is figuring out what our equation looks like, the one that we're going to find. So our equation is going to look like $c \cdot t$ plus $d \cdot t$ squared equals y . So that's what we're looking for. We're looking for a quadratic equation through the origin. Now, if it were just any quadratic equation, then we would have a constant term, but through the origin just means that the constant term is 0.

Now, what's the next step? Next we need to set up a matrix equation. So we need an A , a matrix A . And which matrix is that going to be? Well, let me start with the first coordinate of these three points. And I'm going to put them in the first column of this matrix, 1, 2, -1. And then my second column is going to be the squares of these coordinates, 1, 4, and 1. And my x hat-- that's just c and d . And my b -- this is going to be the second coordinates, 1, 5, and -2.

OK, why did I set this problem up like that? Well, multiply A times x hat. The first coordinate of A times x hat is 1 times c plus 1 times d . It's just the same as plugging in this first point into the left-hand side of this equation.

And similarly, if I took the second point and plugged it in here, I would just get 2 times c plus 4 times d , which is just the same as the second coordinate in the multiplication A times x hat. 2 times c plus 4 times d . OK, and where did the b come from? Well, b came from plugging in these points to the right-hand side. So 1, 5, and -2 are just the y -coordinates of these three points.

So it would be great if we could solve $A x$ hat equals b . But we can't solve $A x$ hat equals b , because there isn't a quadratic equation through the origin that contains these three points. But we need to find the best approximation to that, so that's the same as solving $A x$ hat equals the projection of b onto the column space of A . Because we only have a chance of solving $A x$ hat equals something if it's in the column space of A . And remember from our projections part of the class that this is the same as solving A transpose $A x$ hat equals A transpose b .

So that's what we're really going to do. We're really going to solve $A^T \hat{x} = b$. And now all we have to do is just a computation. So let's do it.

So what is $A^T A$? I'm going to do this kind of quickly, because you should be good at multiplying matrices by now. Oh this-- I have the negative in the wrong place, thank you.

Thanks for that correction. I have the negative backwards there. And what do I get when I multiply these? I'll let you check. This is 6, 8; 8, and 10.

And what is $A^T b$? Well, if you multiply this out, I'll let you check that you get 13 and 19. So this is just some computation. And so what we're solving here is $\begin{bmatrix} 6 & 8 \\ 8 & 10 \end{bmatrix} \hat{x} = \begin{bmatrix} 13 \\ 19 \end{bmatrix}$.

And we remember how to do this just by using elimination. We replace the second row by 3 times the second row minus 4 times the first row. Again, I'm going to do this quickly because you know this from other parts of the class. You similarly change the b vector, and we backsolve to get $d = -5/2$ and $c = 11/2$. Let's plug that in and see-- think you're going to get $11/2$ over 2.

So what's our final equation? Our final equation is $y = 11/2 t - 5/2 t^2$. OK. So this is our best fit quadratic equation through the origin.

Now before I end, let me do a couple things. First, let me go back and review what the key steps were. Whenever you're faced with such a best fit equation, first, you want to see what the general form of the equation is. Next you want to write it in terms of matrices. Write down your matrix A and your vector b. And lastly, you set up your projection equation. And then all you have to do is just a computation.

But it might also be worth noting that these three points certainly aren't on this quadratic equation. For instance, if I plug in 1 here, I don't get (1, 1), I get (1, 3). But that's OK. This is as close as we can do. Thanks.