18.075 Practice Test 2 for Quiz 3

December 3, 2004

Justify your answers. Cross out what is not meant to be part of your solution.

Total number of points: 75.

I. (10 pts) Find the region of convergence of the Frobenius series for the Bessel function $J_p(x)$ $(p \geq 0)$.

II. 1. (6 pts) Locate <u>all</u> singularities of the ODE

$$(\sin x)^2 y'' + x\,y' + (1 - \cos x)y = 0.$$

2. (4 pts) Classify the point $x_0 = 0$ for the ODE of part (1).

III. Consider the ODE

$$xy'' + y = 0.$$

1. (2 pts) Classify the point $x_0 = 0$.

2. (5 pts) Find the indicial exponents s_1 and s_2 for this ODE.

3. (10 pts) Obtain a solution to this ODE by the method of Frobenius for the largest of the two exponents found in part (2).

4. (3 pts) Explain whether you can get an independent solution for the smallest of the two exponents by repeating the procedure of part (3) above.

5. (2 pts) What is the form of the general solution?

6. (8 pts) Obtain a second independent solution to this ODE.

IV. Consider the ODE

$$x^2 y'' + x\,y' + (x^2 - p^2)y = 0, \qquad (p \geq 0).$$

1. (2 pts) Write the general solution to this ODE involving two arbitrary constants c_1 and c_2.

2. (8 pts) Determine c_1 and c_2 so that $y(1) = A$ and $y'(1) = B$, where A and B are given.

3. (10 pts) Determine c_1 and c_2 so that $y(0) = A$, where A is a given finite number, <u>when $p = 0$</u>.

V. (5 pts) Give the general solution to the ODE

$$x^2 y'' + xy' - \left(x^2 + \frac{1}{4}\right)y = 0$$

in terms of elementary functions.