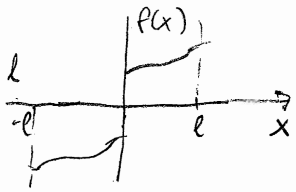
- Fourier sine series: $0<x<l$

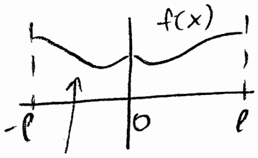


"odd extension" of $f(x)$

$$f(x)=\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right)$$

$$B_n=\frac{2}{l}\int_0^l dx\, f(x)\sin\left(\frac{n\pi x}{l}\right),\quad n=1,2,\ldots$$

- Fourier cosine series: $0<x<l$



"even extension" of $f(x)$

$$f(x)=A_0+\sum_{n=1}^{\infty} A_n\cos\left(\frac{n\pi x}{l}\right)$$

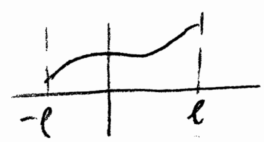
$$A_0=\frac{1}{l}\int_0^l dx\, f(x)$$

average of $f(x)$

careful!

$$A_n=\frac{2}{l}\int_0^l dx\, f(x)\cos\left(\frac{n\pi x}{l}\right),\ n=1,2,\ldots$$

- "Complete" Fourier series: $\boxed{-l<x<l}$



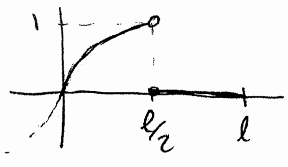
$$f(x)=A_0+\sum_{n=1}^{\infty}\left[A_n\cos\left(\frac{n\pi x}{l}\right)+B_n\sin\left(\frac{n\pi x}{l}\right)\right]$$

$$A_0=\frac{1}{2l}\int_{-l}^{l} dx\, f(x)$$

$$A_n=\frac{1}{l}\int_{-l}^{l} dx\, f(x)\cos\left(\frac{n\pi x}{l}\right)$$

$$B_n=\frac{1}{l}\int_{-l}^{l} dx\, f(x)\sin\left(\frac{n\pi x}{l}\right)$$

ex 49 e $\quad f(x) = \begin{cases} \sin\left(\frac{\pi x}{\ell}\right), & 0<x<\ell/2 \\ 0, & \text{otherwise} \end{cases}$ $\quad (0<x<\ell)$



$$f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{\ell}\right)$$

$$B_n = \frac{2}{\ell}\int_0^{\ell/2} dx\ \sin\left(\frac{\pi x}{\ell}\right)\sin\left(\frac{n\pi x}{\ell}\right)$$

Identity: $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$ $\qquad A = \frac{\pi x}{\ell} \quad B = \frac{n\pi x}{\ell}$

$$\sin\left(\frac{\pi x}{\ell}\right)\sin\left(\frac{n\pi x}{\ell}\right) = \frac{1}{2}\left[\cos\left(\frac{(n-1)\pi x}{\ell}\right) - \cos\left(\frac{(n+1)\pi x}{\ell}\right)\right]$$

$$B_n = \frac{1}{\ell} \int_0^{\ell/2} dx \cos\frac{(n-1)\pi x}{\ell} - \int_0^{\ell/2} dx \cos\frac{(n+1)\pi x}{\ell}$$

$$= \frac{1}{\cancel{\ell}}\left\{\frac{\cancel{\ell}}{(n-1)\pi}\sin\frac{(n-1)\pi x}{\ell}\Big|_{x=0}^{\ell/2} - \frac{\cancel{\ell}}{(n+1)\pi}\sin\frac{(n+1)\pi x}{\ell}\Big|_{x=0}^{\ell/2}\right\} = \frac{1}{\pi}\left\{\frac{1}{n-1}\sin\frac{(n-1)\pi}{2} - \frac{1}{n+1}\sin\frac{(n+1)\pi}{2}\right\}$$

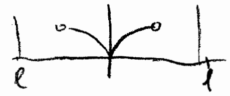
$n \neq 1 \qquad n = 2, 3, \ldots$

$n=1$: $B_1 = \frac{2}{\ell}\int_0^{\ell/2} dx \sin^2\left(\frac{\pi x}{\ell}\right) = \frac{2}{\ell}\int_0^{\ell/2} dx\ \frac{1-\cos\left(2\frac{\pi x}{\ell}\right)}{2}$

$$= \frac{1}{\ell}\left[\frac{\ell}{2} - \cancel{\frac{\ell}{2\pi}\sin\frac{2\pi x}{\ell}\Big|_{x=0}^{\ell/2}}^{\,0}\right] = \frac{1}{2}$$

$$f(x) = \frac{1}{2}\sin\frac{\pi x}{\ell} + \sum_{n=2}^{\infty} B_n \sin\left(\frac{n\pi x}{\ell}\right)$$

ex 50 e $\quad$ cosine



$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{\ell}\right)$ $\qquad A_0 = \frac{1}{\ell}\int_0^{\ell/2} dx \sin\left(\frac{\pi x}{\ell}\right) = \frac{1}{\ell}\cdot\frac{\ell}{\pi} - \cos\frac{\pi x}{\ell}\Big|_{x=0}^{\ell/2} = \frac{1}{\pi}$

$$A_n = \frac{2}{\ell}\int_0^{\ell/2} dx\ \sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{n\pi x}{\ell}\right) =$$

Identity: $\sin(A+\Gamma) + \sin(A-\Gamma) = 2\sin A\cos\Gamma$

$A = \frac{\pi x}{\ell} \qquad \Gamma = \frac{n\pi x}{\ell}$

$$A_n = \frac{1}{\cancel{2}}\frac{\cancel{2}}{\ell}\left\{ \int_0^{\ell/2} dx \, \sin\frac{(n+1)\pi x}{\ell} - \int_0^{\ell/2} dx \, \sin\frac{(n-1)\pi x}{\ell} \right\}$$

$$= \frac{1}{\ell}\left[-\frac{\ell}{(n+1)\pi}\cos\frac{(n+1)\pi x}{\ell}\Big|_0^{\ell/2} + \frac{\ell}{(n-1)\pi}\cos\frac{(n-1)\pi x}{\ell}\Big|_{x=0}^{\ell/2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n-1}\left(\cos\frac{(n-1)\pi}{2} - 1\right) - \frac{1}{n+1}\left(\cos\frac{(n+1)\pi}{2} - 1\right)\right]$$

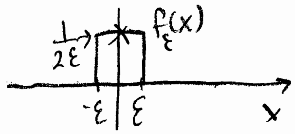
$n = 2, 3, \ldots$

$\underline{n=1}$: $A_1 = \frac{2}{\ell}\int_0^{\ell/2} dx \, \sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi x}{\ell}\right)$

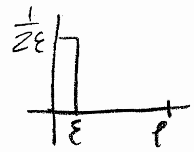
$$= \frac{1}{\cancel{2}}\frac{\cancel{2}}{\ell}\int_0^{\ell/2} dx \sin\left(\frac{2\pi x}{\ell}\right) = \frac{1}{\ell}\left[\frac{\ell}{2\pi}\cdot\left(-\cos\frac{2\pi x}{\ell}\right)\Big|_0^{\ell/2}\right]$$

$$= \frac{1}{2\pi}\cdot 2 = \frac{1}{\pi}$$

$\underline{ex}$ 50f



Expand in complete fourier series. → cos series for



$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{\ell}\right)$$

$$A_0 = \frac{1}{\ell}\int_0^{\varepsilon} dx \, \frac{1}{2\varepsilon} = \frac{1}{2\ell}$$

$$A_n = \frac{2}{\ell}\int_0^{\varepsilon} dx \, \frac{1}{2\varepsilon}\cos\left(\frac{n\pi x}{\ell}\right) = \frac{1}{\ell\varepsilon}\left[\frac{\cancel{\ell}}{n\pi}\sin\left(\frac{n\pi x}{\ell}\right)\Big|_0^{\varepsilon}\right] = \frac{1}{\varepsilon n\pi}\sin\left(\underbrace{\frac{n\pi\varepsilon}{\ell}}_{z}\right)$$

→ $\lim_{\varepsilon\to 0} f_\varepsilon(x) = \delta(x)$ (delta function)

let $z = \frac{n\pi\varepsilon}{\ell}$ $\quad n\pi\varepsilon = \ell z$

$$\lim_{\varepsilon\to 0} A_{n\geq 1}(\varepsilon) = \lim_{z\to 0}\frac{\sin(z)}{\ell z} = \frac{1}{\ell}$$