18.085 Computational Science and Engineering I
Fall 2008

Course Outline: 18.085 Computational Science and Engineering
2008 : The exams normally in class (#11,18,29) will be in the evenings
This will allow more discussion of the topics in the outline

1. Special matrices K,T,B,C
   symmetric, tridiagonal, invertible or singular
   fixed or free boundary conditions

2. Second differences from 1,-2,1
   -u''=f(x) becomes Ku=f
   f= ones, u= quadratic

3. Solving Ku=f
   f=delta, u=ramp
   inverses of K and T: discrete Green's function

4. K=LDL^T from elimination with pivots in D
   K=Q Lambda Q^T with eigenvalues in Lambda
   three-step solution of u'=Ku

5. Eigenfunctions -y''=lambda y
   eigenvectors Ky=(2-2cos(theta))y
   sines, cosines, exponentials in y

6. Positive definite matrices: five tests
   K=A^T A and K=A^T CA are at least semidefinite
   minimizing P=(1/2)u^T K u - u^T f

7. Singular Value Decomposition A=U Sigma V^T
   norms of vectors and matrices
   numerical linear algebra: lu, qr, svd, eig

8. A^T CA for a line of springs
   displacements u from forces f=A^T CAu
   elongation e=Au and balance A^T w=f

9. Oscillation from Mu_{tt}+Ku=0
   exact solution by eigenvectors
   leapfrog and trapezoidal rules

10. Least squares gives A^T Au=A^T b
    solution by orthogonalization A=QR
    weights give A^T CAu=A^T Cb

11. Exam 1 on Lectures 1–9

12 Networks and Incidence Matrix A
    Kirchhoff's Current law A^T w=0
    graph Laplacian A^T A and A^T CA

13. Trusses with 2N displacements
    mechanisms with Au=0
    assembling A and K from each bar

14. Variances and covariances
    optimum weight C=inv(Sigma)
    recursive least squares (Kalman)

15. Continuous A^T CAu=-d/dx(c(x)du/dx)
    integration by parts for adjoint of (d/dx)
    weak form with test functions v(x)

16. Galerkin's trial and test functions give KU=F
    linear finite elements U_1 H_1(x) to U_n H_n(x)
    assembly of matrix K and vector F

17. Quadratic and cubic elements
    beam bending and 4th order problems
    B-splines for interpolation

18. Exam 2 on Lectures 10–16

19. Gradient and divergence
    potential u and stream function s
    equipotentials and streamlines

20. Laplace's equation div(grad u)=0
    polynomial solutions from x+iy
    Cauchy-Riemann equations

21. Finite difference matrix K2D
    fast Poisson solver from sine transform
    odd-even reduction

22. Finite elements: linear in triangles
    assembly of KU=F from element matrices
    boundary conditions and higher order elements

23. Fourier series: sines, cosines, e^{ikx}
    Gibbs phenomenon at jumps
    energy identity and decay rate of coefficients

24. Series solution of the heat equation
    series solution of Laplace's equation on a circle
    delta function and analytic functions

25. Discrete Fourier Transform
    orthogonality of the Fourier matrix
    Fast Fourier Transform

26. Convolution and cyclic convolution
    Fast convolution by Fourier transform
    lowpass and highpass filters; equiripple filters

27. Fourier integrals and energy identity
    input = delta function, output = Green's function
    Heisenberg uncertainty principle and Gaussians

28. Deconvolution and integral equations
    circulant matrices and periodic filters
    autocorrelation and power spectral density

29. Exam 3 on Lectures 19–27

30. Wavelets and scaling functions
    multiresolution and perfect reconstruction
    compressed sensing using l^1 and total variation norms

31. Analytic functions and Cauchy's Theorem
    Chebyshev points and fast transforms
    spectral methods of exponential accuracy