18.085/18.0851 Computational Science and Engineering I	**Summer 2020**

Homework 4

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4.1 Three Coupled Oscillators

Imagine that there are three masses of identical mass $m = 1$ connected by four springs of identical spring constant $c = 1$ on fixed-fixed ends.

Part I: Set up the force balance equation involving the Newton's inertia term and Hooke's law.

Part II: Identify the three normal modes. Which one is the faster than the other two? Can you describe in English what the normal modes are doing to the masses?

You are allowed to do part II analytically or numerically.

- To do it analytically, you will need to repeat the calculation that we went through in class, for three masses
- To do it numerically, you will need to implement the G matrix and perform the eigenvalue decomposition. Make sure to plot the normal modes

4.2 Driven Harmonic Oscillator and Resonance

In the lecture, we claimed that the solution to the driven harmonic oscillator, $mu'' + cu = \cos \omega_0 t$, where ω_0 is very close to the natural frequency $\lambda = \sqrt{\frac{c}{m}}$, is

$$u(t) = \frac{\cos \lambda t - \cos \omega_0 t}{m(\omega_0^2 - \lambda^2)} \tag{4.1}$$

Verify that $u(t)$ defined above indeed satisifes the differential equation of the harmonic oscillator.

Also, show thow that in the limit as $\omega_0 \to \lambda$, $u(t)$ becomes

$$u(t) = \frac{t \sin \omega_0 t}{2m\omega_0} \tag{4.2}$$

4.3 Euler's Formula

Part I: using Euler's identity, prove that

- $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$
- $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$

Part II: using the formulae above, show that for $z = x + iy$

$$sin(z) = \sin(x)\cosh(y) + i\cos(x)\sinh(y) \tag{4.3}$$

4.4 Dynamical System Re-visit I

Solve the following coupled ordinary differential equations

$$\vec{x}' = Ax \tag{4.4}$$

$$\vec{x}(0) = (3, -10)^T \tag{4.5}$$

where

$$A = \begin{bmatrix} 3 & -13 \\ 5 & 1 \end{bmatrix} \tag{4.6}$$

Plot the phase diagrams. Is this a stable or unstable system?

4.5 Dynamical System Re-visit II

Solve the following coupled ordinary differential equations

$$\vec{y}' = By \tag{4.7}$$

$$\vec{y}(0) = (3, -10)^T \tag{4.8}$$

where

$$B = \begin{bmatrix} -2 & 2 \\ -2 & 1 \end{bmatrix} \tag{4.9}$$

Plot the phase diagrams. Is this a stable or unstable system?