18.085/18.0851 Computational Science and Engineering I **Summer 2020**

Homework 2

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2.1 Proof

Show that the centered difference approximation to the first derivative of $f(x)$ is indeed second order

2.2 Numerical PDE

$$\begin{cases} -u''(x) & = \cos(2x) \\ u(0) = u(1) & = 3; \end{cases} \tag{2.1}$$

2.2.1

Solve the system analytically

2.2.2

Implement the solution numerically following the lecture notes. Run it for a mesh size of $1/(N+1)$, for different N. For each N, compute the L^2 norm. Plot the L^2 norm as a function of N and report on the slope.

You should feel free to just modify relevant lines of the dirichlet_code_demo.m file

2.3 Two Delta Functions

Solve the following differential equations

$$\begin{cases} -u''(x) & = \delta(x-1/4) - 4\delta(x-1/2) \\ u(0) = u(1) & = 0; \end{cases} \tag{2.2}$$

2.4 Orthogonality and Symmetricality

Find all 2-by-2 matrices that are orthogonal and also symmetric. Which two numbers can be eigenvalues of these matrices?

2.5 Matrix Diagonalization and ODE

Solve the following systems of ordinary differential equations and plot the phase diagrams

$$\begin{cases} u' = 6u - v \\ v' = 2u + 3v \\ u(0,0) = (1,-1) \end{cases} \tag{2.3}$$

$$\begin{cases} u' = u + 2v \\ v' = 3u + v \\ u(0,0) = (1,11) \end{cases} \tag{2.4}$$

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