18.091: Lab 2

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1 Introduction and Procedure

The goal of this lab project is to relate the rate at which orbits of several functions converge to their respective fixed points to the value of first derivative at the fixed points. We will accomplish this by calculating the first 10,100 iterations of twelve functions using the Java applet provided on Professor Devaney's website and finding the first iteration k such that $d(f^k(x), p) \leq 0.001$, where p is the fixed point.

2 Results

In the following table we list $f(x)$, the function; p, the fixed point; $f'(x)$, the absolute value of the first derivative; n, the number of iterations required for the orbit of 0.2 to come within 0.001 of the fixed point; and whether p is attracting, $|f'(x)| \leq 0$, or neutral, $|f'(x)| = 1$.

$f(x)$	p	n	$f'(x)$	*Classification*
$x^2 + 0.25$	$1/2$	991	1	*neutral*
x^2	0	3	0	*attracting*
$x^2 - 0.24$	$-1/5$	1	$-2/5$	*attracting*
$x^2 - 0.75$	$-1/2$	10000	-1	*neutral*
$0.4 \cdot x(1-x)$	0	6	$2/5$	*attracting*
$x(1-x)$	0	990	1	*neutral*
$1.6 \cdot x(1-x)$	$3/8$	8	$2/5$	*attracting*
$2 \cdot x(1-x)$	$1/2$	4	0	*attracting*
$2.4 \cdot x(1-x)$	$7/12$	5	$-2/5$	*attracting*
$3 \cdot x(1-x)$	$2/3$	10000	-1	*neutral*
$0.4 \sin x$	0	6	$2/5$	*attracting*
$\sin x$	0	10000	1	*neutral*

As we might expect, orbits converge relatively quickly to attracting fixed points because orbits converge exponentially to p as seen on page 44 of the text. In order to understand the behaviour of the neutral fixed points, we must further consider the second and third derivatives of the function to determine the concavity of $f(x)$.

If $f'(p) = -1$, the function's slope is perpendicular to $y = x$, which contains all fixed points of $f(x)$. So, the orbit of x is not moving directly toward the fixed point; instead, the orbit spirals slowly towards the fixed point. If $f'(p) = 1$, then the function's slope is parallel to $y = x$ at the fixed point and we can use the higher order derivatives to see if $f(x)$ is concave up or down; however, if concave up or down the orbit will be bounded by $y = x$ and $f(x)$ and will thus converge more quickly than $f'(p) = -1$. Despite having first derivative of 1 at the fixed point, the Sine function does not behave in this manner. The fixed point p is an inflection point because it's second derivative is 0 and its third derivative is -1. On the left side of the inflection point orbits go to $-\infty$ and therefore do not approach this fixed point whatsoever. Orbits beginning to the right of the inflection point approach p very slowly because at p $f(x)$ is nearly a line.