

# 18.091: Lab 3

Laura Hajj

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## 1 Introduction

In this experiment, we will examine the so-called windows of relatively empty space in the orbit diagrams of the quadratic map,  $Q_c(x) = x^2 + c$  and the logistic map,  $f_\lambda(x) = \lambda x(1 - x)$ . We will compare and contrast a number of smaller windows in these mappings and determine the periods of these windows. Here period refers to the period of the attracting orbit before it undergoes period doubling.

## 2 Procedure and Data

Using the online java applet, we create generations of magnifications of the orbit diagram by magnifying the orbit diagram iteratively between each successive pair of windows. That is, generation 1 is the two largest windows of the orbit diagram for the map on normal range and domain and generation 2 is those two windows plus the two largest windows in the magnification of generation one between generation 1's two windows. For each of our mappings, we create a chart listing the periods of the largest windows of each successive generation with each row left to right represents increasing values of  $c$  or  $\lambda$ .

Quadratic Function,  $Q_c(x) = x^2 + c$ :

<b>Generation 1:</b>	3								1
<b>Generation 2:</b>	3		5			6			1
<b>Generation 3:</b>	3	7	5	7	8	6	10	12	1
<b>Generation 4*:</b>	6	14	10	14	16	12	20	24	1
<b>Generation 5*:</b>	12	28	20	28	32	24	40	48	1

\*Generation 4 lists only the periods of windows between windows of period 6 and 1 in Generation 3

\*Generation 5's values are predicted from a magnification between Generation 4's windows of period 12 and 1

Logistic Family,  $f_\lambda(x) = \lambda x(1 - x)$ :

<b>Generation 1:</b>	1							3	
<b>Generation 2:</b>	1		6			5		3	
<b>Generation 3:</b>	1	12	10	6	8	7	5	7	3
<b>Generation 4*:</b>	1	24	20	12	16	14	10	14	6
<b>Generation 5*:</b>	1	48	40	24	32	28	20	28	12

\*Generation 4 lists only the periods of windows between windows of period 6 and 1 in Generation 3

★ Generation 5's values are predicted from a magnification between Generation 4's windows of period 12 and 1

### 3 Analysis

We immediately see that the periods of the windows for both orbit diagrams are the same for each generation only in reverse order, which we might expect from the graphs of the orbit diagrams. We certainly see a pattern in our charts. As the generations progress, we notice that the successive magnifications produce windows with periods double that of the the entire generation before it because of self-similarity in our functions. We also see that that the period-1 window is constant throughout all generations.