18.091: Lab 1

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1 Introduction

The purpose of this lab is to demonstrate that computer-generated orbits of seemingly simple functions can be deceptive or completely inaccurate. We will use Mathematica to record the orbits for three different functions and to examine the subsequent graphs.

2 Procedure and Results

In each of the three following subsections, we will examine the orbits of a different function for ten different seeds. For each seed iterated on the given function, we compute the first 100 points of the orbit and then report if the orbit was fixed (F), eventually fixed (EF) periodic (P), eventually periodic (EP), or without visible pattern (NVP).

2.1 The Quadratic Map $F(x) = x^2 - 2$

For the Quadratic Function with $c = -2$, we will choose initial seeds such that $x_0 \in (-2, 2)$.

x_0	behavior
-1.5	NVP
-0.75	NVP
-0.6	NVP
-0.5	NVP
0.1	NVP
0.35	NVP
0.5	NVP
0.75	NVP
1.1	NVP
1.5	NVP
0	EF at 2
1	EF at -1

So, this function appears chaotic with fixed points after iteration. That is, each of the non-fixed orbits appears to visit any subinterval of $(-2, 2)$ unpredictably.

2.2 The Quadratic Function, $G(x) = x^2 - 3$

We iterate this function using the same seeds as the similar previous function.

x_0	behavior
-1.5	∞
-0.75	∞
-0.6	∞
-0.5	∞
0.1	∞
0.35	∞
0.5	∞
0.75	∞
1.1	∞
1.5	∞
2	EP

This function has orbitals that grow to infinity exponentially. That is, $G^n(x) \to x^{2^n}$. However, it also has fixed and periodic points.

2.3 The Doubling function $D(x) \equiv 2x \pmod 1$

For the doubling function our seeds are chosen in the interval $[0, 1)$.

x_0	behavior
$\frac{1}{17}$	P
0.0582	EF at 0, $n = 52$
$\frac{1}{21}$	P
0.0467	EF at 0, $n = 57$
0.1	EF at 0, $n = 55$
$\frac{1}{9}$	P
$\frac{1}{5}$	P
0.35	EF at 0, $n = 53$
0.5	EF at 0, $n = 1$
0.75	EF at 0, $n = 2$
0	F

Here we see a mixture of behaviors, which is seemingly apparent on the accuracy of the decimal approximation near certain rational numbers.

3 Conclusion

In the first example, we clearly see chaotic behavior. The second function, although similar to the first, displays different behavior. Most of $G(x)$ orbits tend to infinity, except those that are periodic or fixed. The real interest of this lab lies in the third function, where we can easily see how many computer programs' rounding error would cause us to miss the periodicity of orbits resulting from certain rational seeds. We don't see these problems in Mathematica because it utilizes exact rational arithmetic. However, many other programs round decimial approximations to $\sum_{i=0}^{\infty} \frac{b_i}{b^i}$ and numbers of this form always tend to 0 under iteration by $D(x)$.