# 18.091 Lab 3 Rewrite 

Wes McKinney - 11 May 2005

## 1 Experiment

The objective of this experiment is to determine the periods of the various width period $k$ windows that appear in the orbit diagrams of the quadratic map, $Q_{c}(x)=x^{2}+c$, and the logistic family, $f_{\lambda}(x)=\lambda x(1-x)$. Along these lines, we compare the orbit windows for these functions. The online java applet plotting the orbit diagrams of these functions was used, which allowed magnification of the desired orbit windows to determine the periods. The results have been put in table format, organized by "generation", which roughly translates to the width of the orbit window, or the degree of visibility of an orbit window at a particular magnification. As it turns out, on examination of these orbit windows, the distinction of width and visibility is very clear between generations.

On comparing the periods for the quadratic family and the logistic family, we find that the periods are the same, though in reverse order, which agrees with the appearance of their orbit diagrams (which open in opposite directions). Additionally, we observe that most of the numbers for Generation 4 are twice those of the number appearing directly above them in the chart for Generation 3. More investigation would be needed to determine is this is a continuing trend, but a reasonable conjecture to make would be that the period-1 end of the list for Generation 5 will also be double that of Generation 4. We list this prediction for Generation 5 as part of the tables for both of the functions.

| Generation |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 3 |  |  |  |  |  |  |  | 1 |
| $\mathbf{2}$ | 3 |  | 5 |  |  | 6 |  |  | 1 |
| $\mathbf{3}$ | 3 | 7 | 5 | 7 | 8 | 6 | 10 | 12 | 1 |
| $\mathbf{4}$ | 6 | 14 | 10 | 14 | 16 | 12 | 20 | 24 | 1 |
| $\mathbf{5}$ (pred) | 12 | 28 | 20 | 28 | 32 | 24 | 40 | 48 | 1 |

Function: $x^{2}+c$. These are the orbit windows for the quadratic map, ordered by increasing $c$. For Generation 2, the two largest windows between the period-1 and period-3 windows are 5 and 6. In Generations 3 and 4, we look between the windows of the previous generation to determine the order of the smaller windows. In Generation 4, only those windows between the period-6 and period-1 windows are counted. The prediction for Generation 5 are those windows between the period-12 window and the period-1 window from Generation 4.

| Generation |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 1 |  |  |  |  |  |  |  | 3 |
| $\mathbf{2}$ | 1 |  |  | 6 |  |  | 5 |  | 3 |
| $\mathbf{3}$ | 1 | 12 | 10 | 6 | 8 | 7 | 5 | 7 | 3 |
| $\mathbf{4}$ | 1 | 24 | 20 | 12 | 16 | 14 | 10 | 14 | 6 |
| $\mathbf{5}$ (pred) | 1 | 48 | 40 | 24 | 32 | 28 | 20 | 28 | 12 |

Function: $\lambda x(1-x)$. Analogous table for the logistic family of functions. Notice that the values are the same as those in the previous table, though reflected.

