

## 18.100A Fall 2012: Assignment 25

As before, list collaborators, if any; it's illegal to consult assignment solutions from previous semesters.

**Reading: (Mon.) 27.1-.2; 27.3** (statement only) Improper integrals depending on a parameter: uniform convergence, M-test analog; continuity. (Problems 1-4)

**(Wed.) 27.3-.5** Continuity, Fubini, Differentiability, Laplace transform apps. (Problems 5-7)

1. (1) Work 27.2/2, (The integral should have a  $dt$ .) This uses the the M-test analog. In using it, be careful about values of  $t$  near 0.)
2. (2) Using the book's definition of *exponential type*: (8), p.389,
  - a) show the function  $h(t) = t^m$ ,  $m > 0$ , is of exponential type, and give the possible value(s) of  $k$  in (8);
  - b) tell for what  $x$ -values its Laplace transform converges, and for what  $x$ -values it converges uniformly; give the reasoning from scratch by using theorems, i.e., don't just quote the results in Example 27.2B, though you can consult it.
3. (2: 1, .5, .5) For the integral in 27.3/1, answer the following.
  - a) On what  $x$ -intervals does it converge uniformly?
  - b) What is the largest  $x$ -interval on which it converges (pointwise)?
  - c) Prove it is continuous for  $x > 0$  (do not use **7.** below).
4. (2) The Gamma function is defined in **21.3** to be  $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ .  
Prove it is continuous for  $x \geq 1$ ; use theorems. (It is actually continuous for  $x > 0$ .)
5. (3: 1.5, 1.5) a) Work 27.3/2      b) Work 27.4/2
6. (2) Work 27.5/2, without citing the results in Example 27.5.  
Treat this like a Question, whose answer is Example 27.5. You can study the example first, but let a little time elapse and try to give the argument for the specific function  $\sin t$  without looking back at the example.
7. (3; 2,1) Work 27.4/3 (it continues 27.3/1; don't repeat what you did in **3** above):
  - a) show  $y(x)$  is differentiable for  $x > 0$ ; (cite theorems, verify hypotheses)
  - b) show  $y(x)$  satisfies the differential equation, for  $x > 0$ .

*On Friday and Monday we will cover Chapter 23 – Sets of measure zero and the Lebesgue Integral. This is interesting, but optional material; there will be a question on it on the final exam, but there will be enough choice so you won't have to answer it. For those who would like to learn it, I'll post on Friday an assignment 26 on Chapter 23 (not to be handed in), with solutions available in class Monday and sent out by e-mail afterwards.*

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