Exercises given with a numbering are from Basic Analysis: Introduction to Real Analysis (Vol I) by J. Lebl.

Reading Sections 4.3, The Riemann Integral lecture notes

## Exercises

1. Prove that the polynomial equation $\frac{x^{1121}}{1121}+\frac{x^{2021}}{2021}+x+1=0$ has exactly one real root.
2. Compute the fourth Taylor polynomial for:
(a) $f(x)=\sin x$ at $x=0$.
(b) $f(x)=\frac{1}{1-x}$ at $x=-1$.
3. Compute:
(a)

$$
\lim _{x \rightarrow 0} \frac{x-\sin x}{x^{3}}
$$

(b)

$$
\lim _{x \rightarrow \frac{\pi}{2}} \frac{1-\sin x}{\left(x-\frac{\pi}{2}\right)^{2}}
$$

4. Suppose that $f:(a, b) \rightarrow \mathbb{R}$ is three times continuously differentiable, $c \in(a, b)$, $f^{\prime}(c)=f^{\prime \prime}(c)=0$ and $f^{\prime \prime \prime}(c)>0$. Prove that $f$ has a neither a local maximum nor a local minimum at $c$.
5. Let $a<b$, and define a sequence of tagged partitions by

$$
\begin{aligned}
\underline{x}^{(r)} & :=\left\{a+(b-a) \frac{k}{r}: k=0, \ldots, r\right\}, \\
\underline{\xi}^{(r)} & :=\left\{a+(b-a) \frac{k}{r}: k=1, \ldots, r\right\} .
\end{aligned}
$$

(a) Compute $\left\|\underline{x}^{(r)}\right\|$.
(b) Let $f(x)=\alpha x+\beta$. Prove that

$$
\lim _{r \rightarrow \infty} S_{f}\left(\underline{x}^{(r)}, \underline{\xi}^{(r)}\right)=\alpha \frac{b^{2}-a^{2}}{2}+\beta(b-a) .
$$

You may not use the fundamental theorem of calculus.

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### 18.100A / 18.1001 Real Analysis

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