Exercises given with a numbering are from *Basic Analysis: Introduction to Real Analysis (Vol I)* by J. Lebl.

Reading Sections 3.1, 3.2

Exercises

1. Exercise 3.1.3

2. Let

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q}, \\ 2x & \text{if } x \notin \mathbb{Q}. \end{cases}$$

   Prove that f is continuous at $x = 0$ and discontinuous at $x = 1$.

3. Exercise 3.2.11

4. Exercise 3.2.14

5. Let $f : \mathbb{R} \to \mathbb{R}$. Recall that if $U \subset \mathbb{R}$, the *inverse image* of U is the set

$$f^{-1}(U) := \{x \in \mathbb{R} : f(x) \in U\}.$$

   Prove that f is continuous if and only if for every open set $U \subset \mathbb{R}$, $f^{-1}(U)$ is open.

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