Exercises given with a numbering are from *Basic Analysis: Introduction to Real Analysis (Vol I)* by J. Lebl.

**Reading** Sections 3.1, 3.2

**Exercises**

1. Exercise 3.1.3
2. Let

\[
f(x) = \begin{cases} 
0 & \text{if } x \in \mathbb{Q}, \\
2x & \text{if } x \notin \mathbb{Q}.
\end{cases}
\]

Prove that $f$ is continuous at $x = 0$ and discontinuous at $x = 1$.

3. Exercise 3.2.11
4. Exercise 3.2.14
5. Let $f : \mathbb{R} \to \mathbb{R}$. Recall that if $U \subset \mathbb{R}$, the *inverse image* of $U$ is the set

\[
f^{-1}(U) := \{ x \in \mathbb{R} : f(x) \in U \}.
\]

Prove that $f$ is continuous if and only if for every open set $U \subset \mathbb{R}$, $f^{-1}(U)$ is open.
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