18.100**C**
Fall 2012

Problem Set 3

1. Let (X, d) be a metric space. Show that $d'(x, y) = \sqrt{d(x, y)}$ is also a metric on X, and that the open sets for d' are the same as the open sets for d. (2 points)
2. Consider $\mathbb{R}$ with the standard metric. Let $E \subset \mathbb{R}$ be a subset which has no limit points. Show that E is at most countable. (3 points)
3. Problem 24 from page 45 (the word "separable" is explained in Problem 22 on the same page).
   When writing the answer for this problem, please pay particular attention to completeness of the argument; and to structure, clarity and legibility of writing. From this week onwards, **LaTeX is required** *(for this problem only). Your answer will be assessed by the grader for correctness, and then again by the recitation instructor for quality of exposition.* (4 points)
4. Let (X, d) be a compact metric space, and $f : X \to X$ a map such that $d(f(x), f(y)) < d(x, y)$ for all $x \neq y$. Prove that there exists a point x such that $f(x) = x$. Hint: how small can $d(x, f(x))$ get? Comment: this is an example of a fixed point theorem, very popular in various sciences for showing the existence of equilibria and such. (5 points)

Total: $2 + 3 + 4 + 5 = 14$ points.

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