18.200 Homework 7

Instructions: Indicate your recitation and collaborators or state that you worked only on your own. In any case, you must write up your own proofs.

1. Consider sets $S_1, S_2, \cdots, S_n \subset [n]$ and suppose we choose a random function $f : [n] \to \{\pm 1\}$. That is for each integer i between 1 and n we flip a fair coin to decide if $f(i) = +1$ or else $f(i) = -1$.

   (a) Show that for any fixed set S_j, with probability $1 - 1/n^2$ we have that

   $$\sum_{i \in S_j} f(i) \ \leq C\sqrt{n \log n}$$

   for some constant C. This is called the discrepancy of the set S_j.

   (b) Use your answer to the previous part to argue that there is a function $f : [n] \to \{\pm 1\}$ so that every set S_j has discrepancy at most $C\sqrt{n \log n}$

2. Transform the following linear program into an equivalent linear program in standard form ($\max\{c^T x : Ax = b, x \geq 0\}$) and canonical form ($\max\{c^T x : Ax \leq b, x \geq 0\}$).

   $$\begin{aligned} \min \quad & x_1 - x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \geq 3 \\ & 3x_1 - x_2 \leq 7 \\ & x_1 \geq 0, \ x_2 \lesseqgtr 0. \end{aligned}$$

3. Write the dual of the linear program in Problem 2.

4. Suppose we are packing a bag that has total volume V and can carry a total weight of W. Each item of type i takes up v_i space and has weight w_i and has value c_i for our trip. Write a linear program to express the problem of how to pack our bag to maximize the total value, subject to the constraint that the total volume is at most V and the total weight is at most W. You may assume that the items are infinitely divisible, so you can take any fractional value of them you'd like.