18.200 Homework 8

Instructions: Indicate your recitation and collaborators or state that you worked only on your own. In any case, you must write up your own proofs.

1. Construct a pair of dual linear programs such that both the primal and the dual are infeasible (i.e., the feasibility region is empty).
   Hint: It suffices to have only two variables for both the primal and the dual.

2. There is a children's game "rock, paper, scissors," which is popular in the United States
   In it, two players simultaneously choose a hand signal for rock, paper, or scissors.
   Players win according to the following rules:

   Paper beats rock (paper *covers* rock),
   Scissors beats paper (scissors *cut* paper).
   Rock beats scissors (rock *smashes* scissors).

   Clearly, the optimal strategy in this game (where the payoff for winning is 1 and the payoff for losing is -1) is to choose each possibility (R,P,S) with probability $\frac{1}{3}$.

   There is a school where the kids playing this are somewhat sadistic, When somebody gets rock and their opponent gets scissors, the rock player smashes the scissors player's fingers as hard as they can. Thus, the players try to avoid having their fingers smashed. Simultaneously, they take satisfaction in smashing the other player's fingers. Let's assume that the payoff matrix for the game is now:

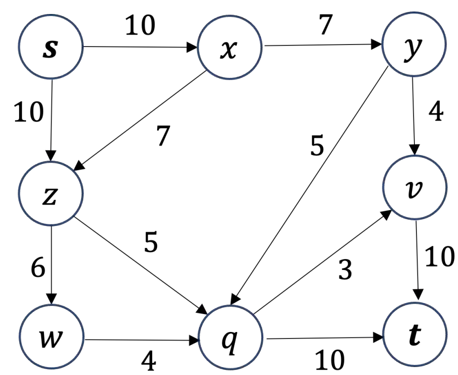
$$C = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -1 \\ -2 & 1 & 0 \end{pmatrix}$$

   (a) (Optional) Before doing any calculations, try to guess which actions will have increased probability and which will have decreased probability with the new payoff matrix C.

   (b) Find the optimal strategy with the payoff matrix C.

   Note: You can solve this problem using symmetry and complementary slackness.

3. Consider the flow network presented in the figure below, with source s and target t. The capacity of each edge appears next to the edge. Find a maximum s-t flow and a minimum s-t cut in this network.



4. Let x, y, z be three nodes in some flow network, and suppose that there exists a flow of value C between x and y in the network, and a flow of value C between y and z in the network. Prove that there is a flow of value C between x and z in the network.