Proofs in Mathematics and Proof Strategies

18.200

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February 7, 2023

What do mathematicians mean by **proof**?

Proof is “a logical argument that establishes the truth of a statement beyond any doubt. A proof consists of a finite chain of steps, each one of them a logical consequence of the previous one.”

Cupillari, A. (2005). *The Nuts and Bolts of Proofs: An Introduction to Mathematical Proofs*. Academic Press.

At some point in the history of the world, no one had ever written a **proof**.

Why did they do it?
Do you think it was a **good** idea?
Why or why not?

Proofs are the primary form of **argumentation** in mathematics.

We can **model** the **role** that **proofs** play in **building mathematics knowledge** by looking at the **reasoning** of mathematics.

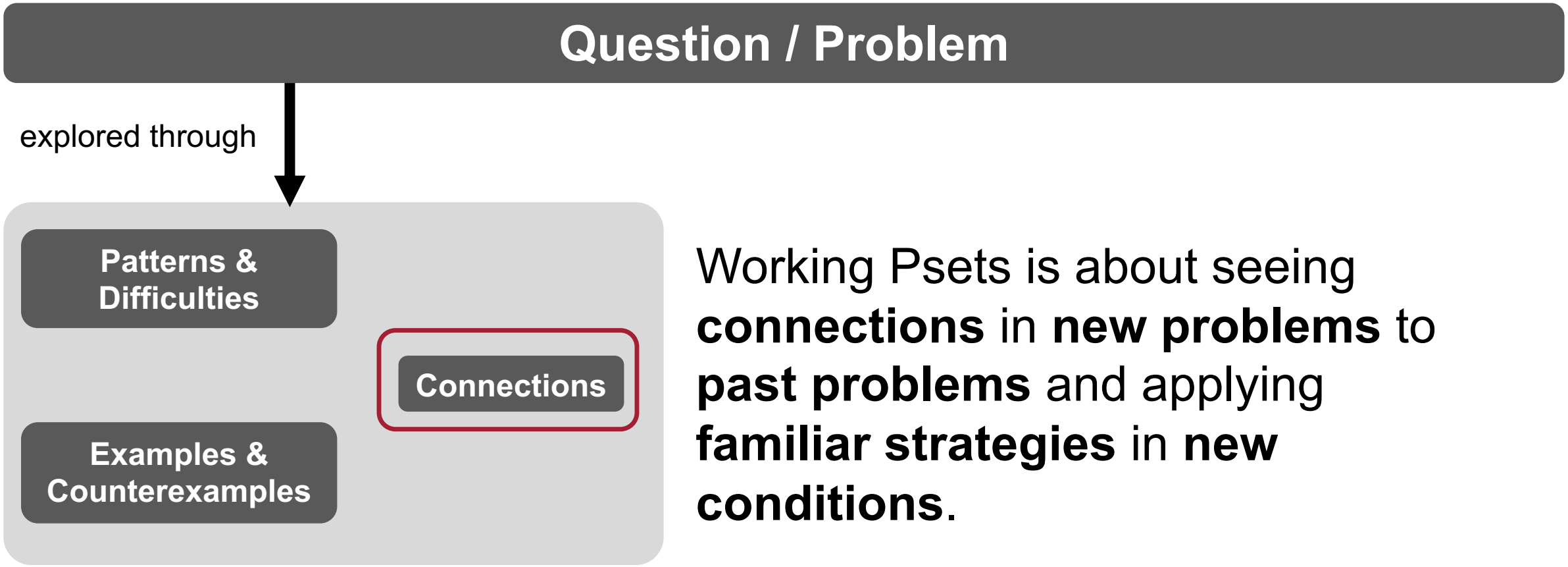
Mathematics Reasoning Diagram

Focus on Proofs

Question / Problem

Mathematics Reasoning Diagram

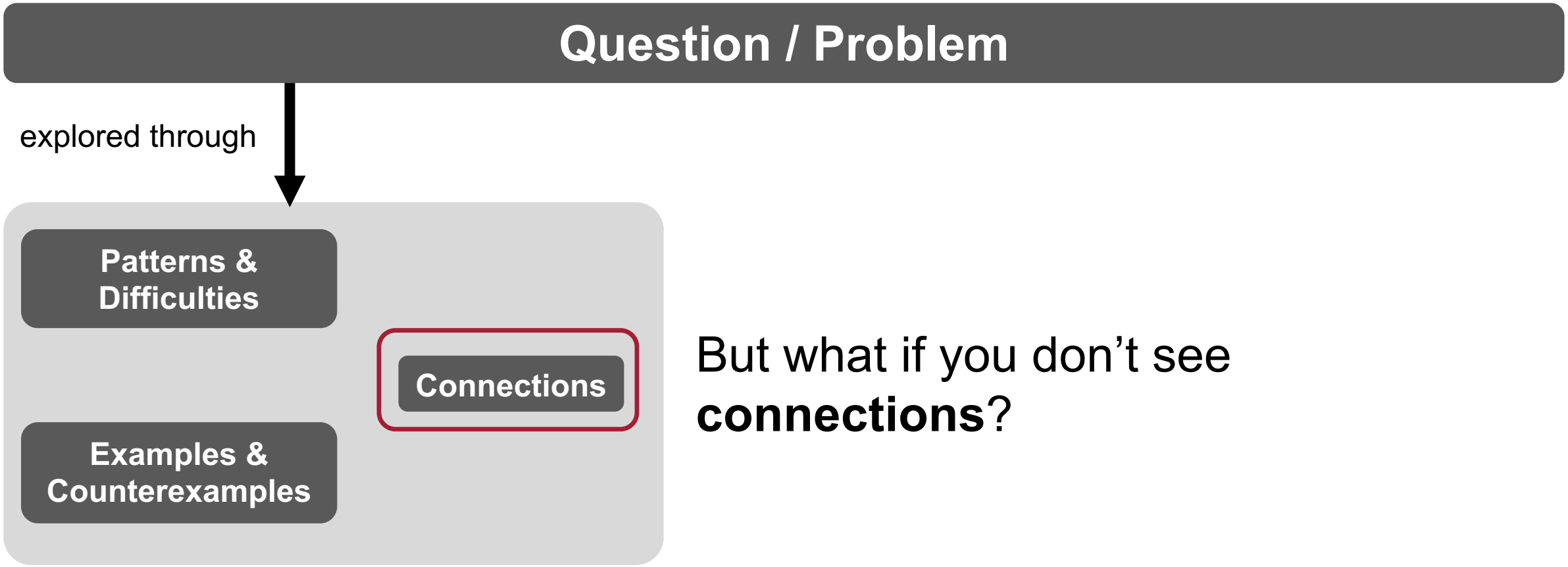
Focus on Proofs



Working Psets is about seeing **connections** in **new problems** to **past problems** and applying **familiar strategies** in **new conditions**.

Mathematics Reasoning Diagram

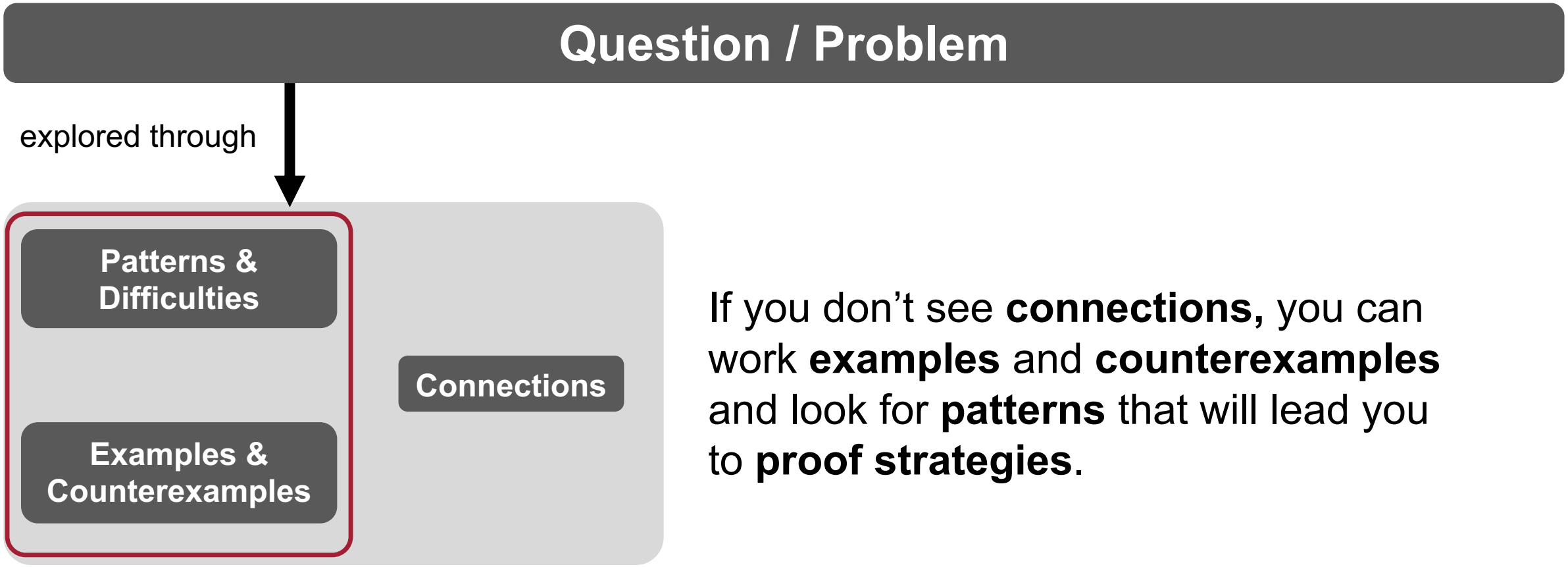
Focus on Proofs



But what if you don't see **connections**?

Mathematics Reasoning Diagram

This slide will help with Recitation 1*

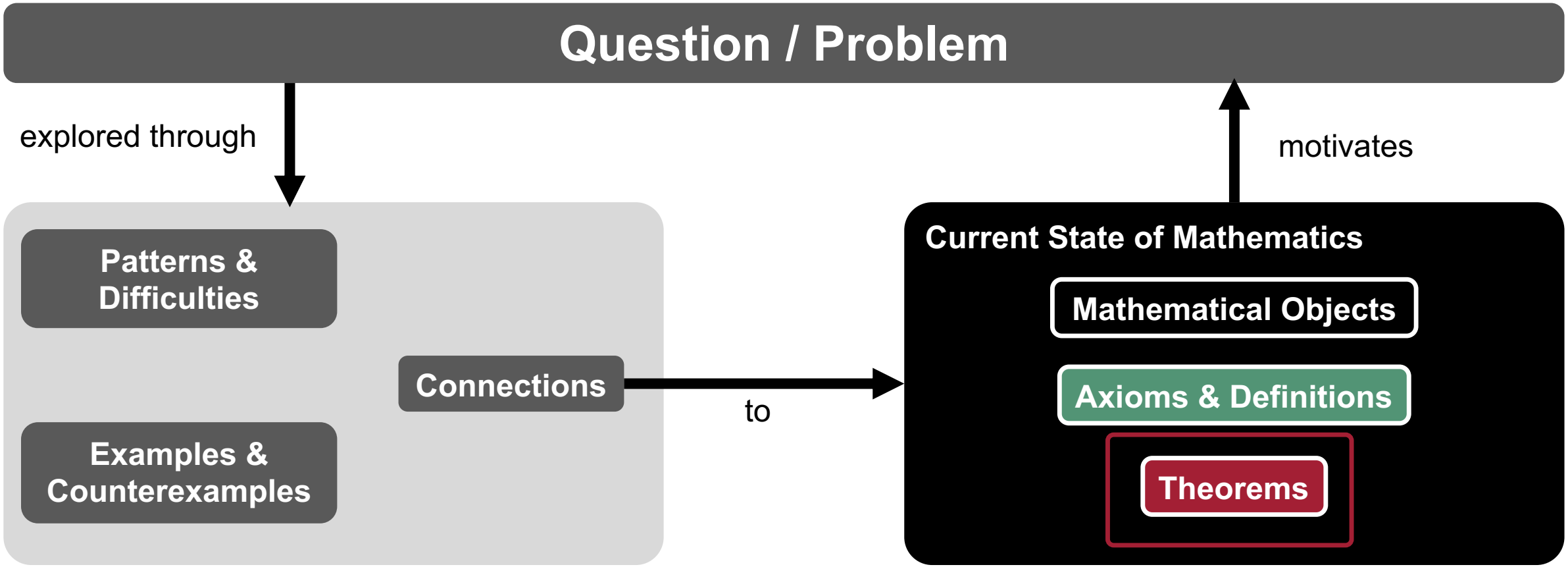


If you don't see **connections,** you can work **examples** and **counterexamples** and look for **patterns** that will lead you to **proof strategies**.

*and all the rest of your math problem-solving

Mathematics Reasoning Diagram

Focus on Proofs



Working through **problems** leads to **development** of **knowledge** in **mathematics**.
This includes **theorems**, which need to be **proved**.

What is a **Theorem**?

A **Theorem*** is a statement that gives all the conditions, or **hypotheses**, under which its concluding claim, or **conclusion(s)**, can be shown to be true by "a finite chain of steps, each one of them a logical consequence of the previous one."

*We will nuance this definition in later recitations.

Cupillari, A. (2005). *The Nuts and Bolts of Proofs: An Introduction to Mathematical Proofs*. Academic Press.

What is a Theorem?

Theorem = **hypotheses** + **conclusion(s)**

So what questions might arise if all we have is:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

What is a **Theorem**?

Theorem = **hypotheses** + **conclusion(s)**

How about now?

Theorem (Gauss Sum). *For all natural* $n \geq 1$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} .$$

What is a **Theorem**?

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What is a Theorem?

Theorem = **hypotheses** + **conclusion(s)**

How about now?

Sentence starts with word, not symbol.

Theorem (Gauss Sum). *For all natural* $n \geq 1$,

Theorem label

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

Always have word on one side of punctuation.

Punctuate around the math symbols.

What is a **Theorem**?

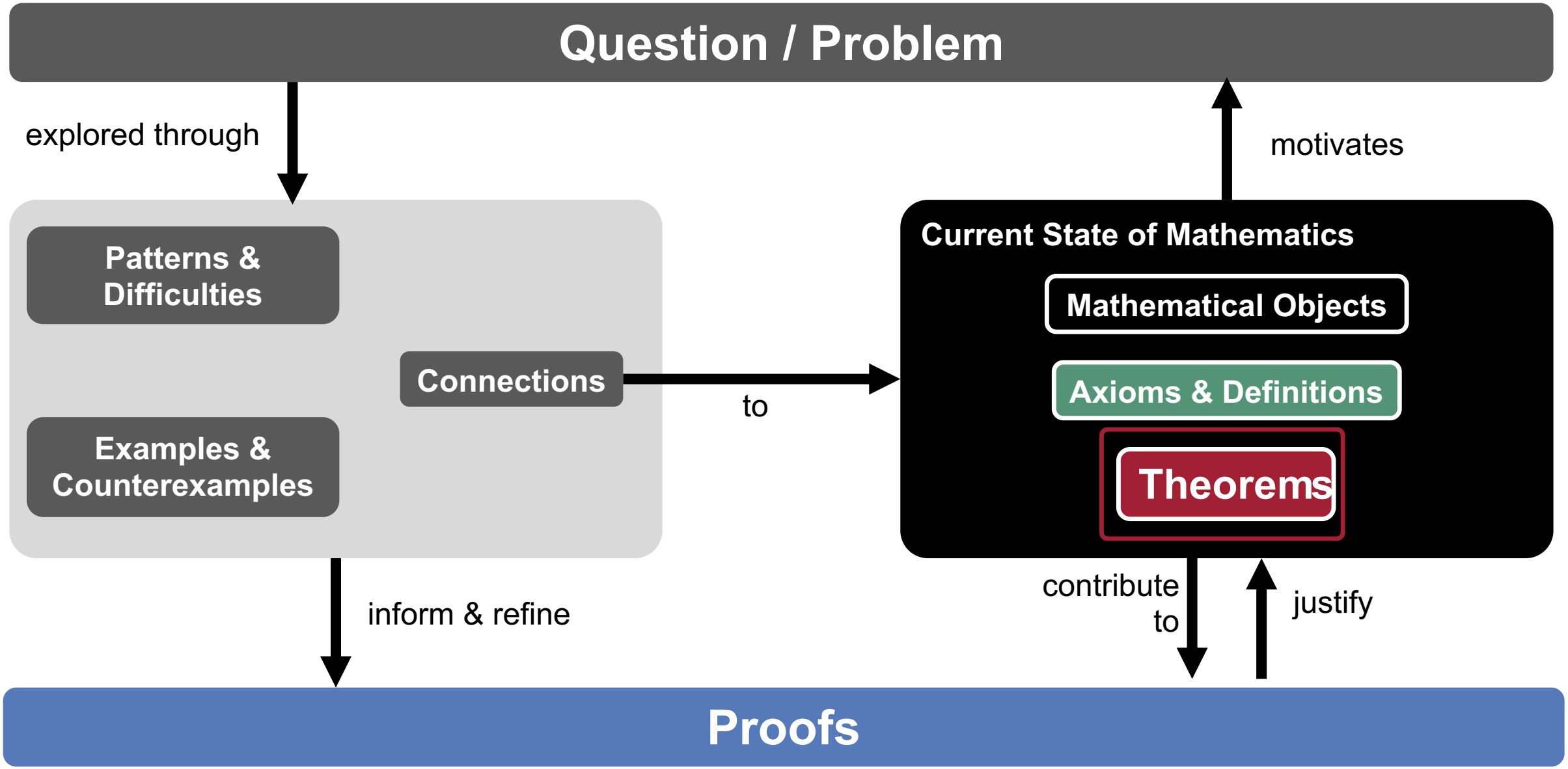
Theorem = **hypotheses** + **conclusion(s)**

shown to be true by

Proofs

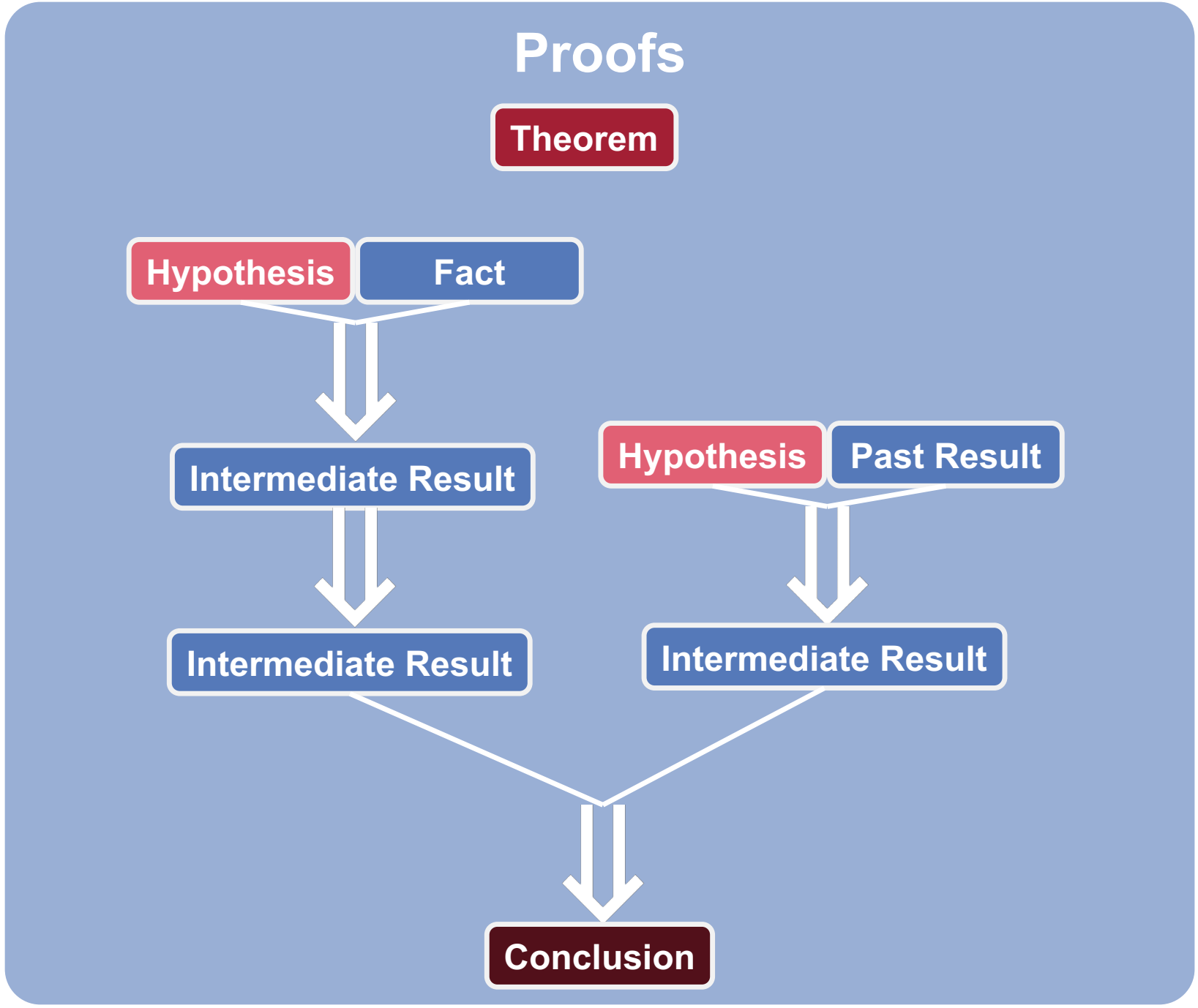
Mathematics Reasoning Diagram

Focus on Proofs



Proofs

Over time, **mathematicians** developed **conventions** for **proofs** so mathematicians could **accomplish** their **goals**.



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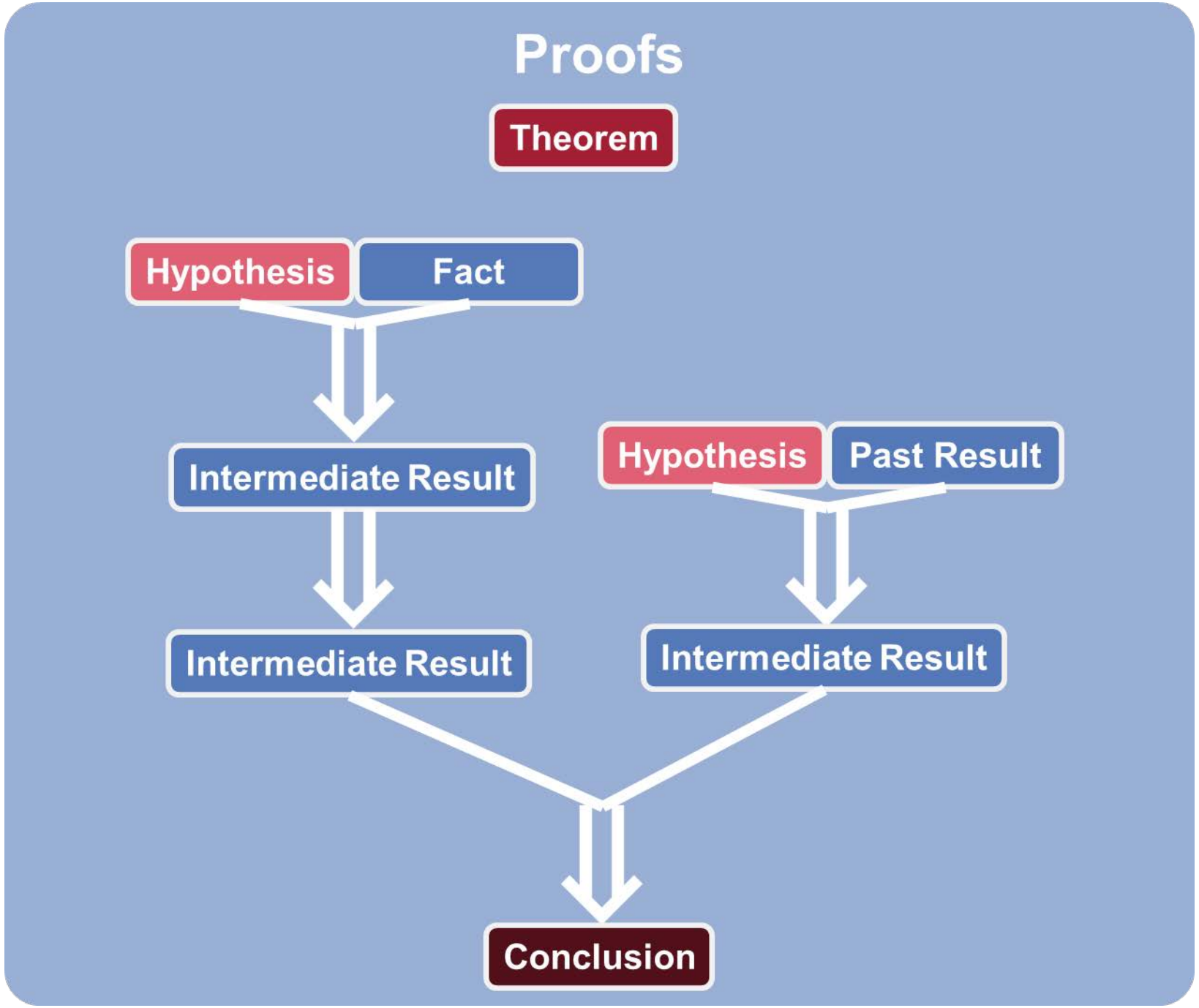
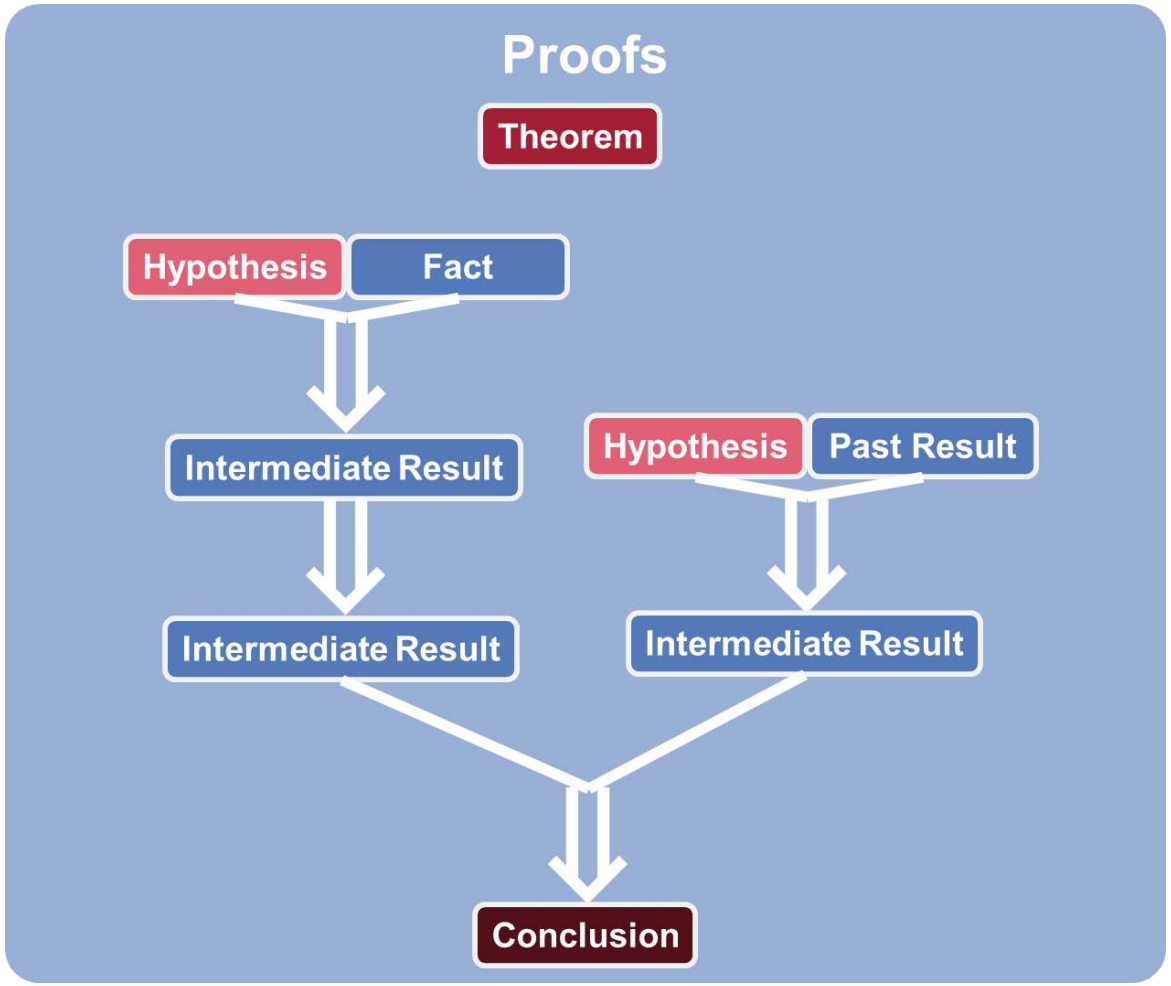


Whenever people develop a **conventionalized way** of accomplishing a **recurring communication goal**, we call this a **genre**.

What does all this look like when written up?

This slide will help you with your Pset 1*

*and all the rest of your writing in math



Theorem-Proof Formatting

Theorem 1. *In a right-angled triangle, with side lengths a, b, and c, where c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.*

Proof. Given a triangle with side lengths a, b, and c, where the side with length c is the hypotenuse, construct a square with side length $a + b$ from four copies of this triangle. The total area of this square is

$$A = (a + b)^2.$$

This total area can also be described as the area of the four copies for the original triangle and an interior square of side length c, so

$$A = c^2 + 4\left(\frac{ab}{2}\right).$$

Setting these two calculations of the area equal and then simplifying both sides yields

$$\begin{aligned}(a + b)^2 &= c^2 + 4\left(\frac{ab}{2}\right)\\ a^2 + 2ab + b^2 &= c^2 + 2ab\\ a^2 + b^2 &= c^2.\end{aligned}$$

□

How do you get theorem-proof formatting?

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LaTeX

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```
\begin{theorem}
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$c$ is the length of the hypotenuse, then $a^2+b^2=c^2$.
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 \end{align*}
\end{proof}
```
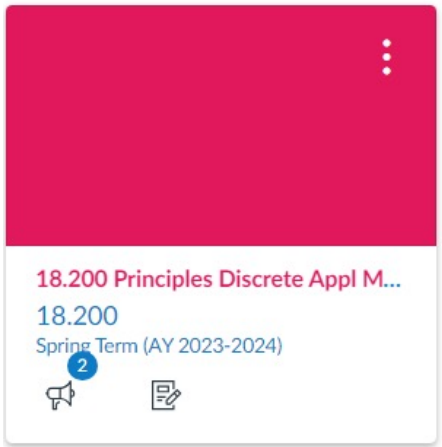

How do I learn LaTeX ?

This slide will help you with your Pset 1*
***and all the rest of your writing in math**

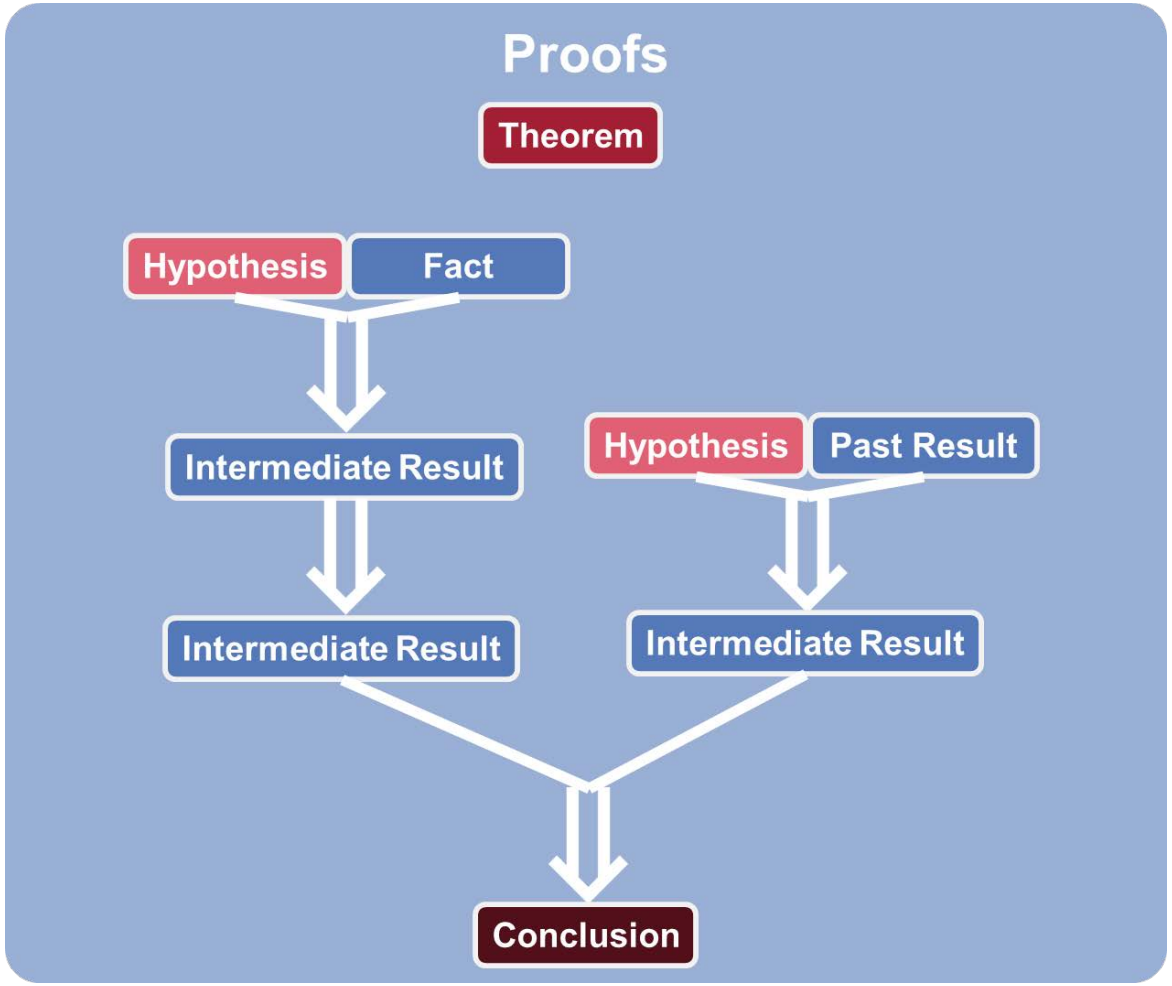
Canvas! (+ Overleaf)



Overleaf

Link to Homework Template for Overleaf

Link to Homework Template for desktop LaTeX editor



Helping you learn how to write in the **proof genre effectively** is my **goal** in **communication** this semester.

My contact info
for writing office hours
is in the syllabus

18.200 Principles of Discrete Applied Mathematics

Spring 2024