Problems for The 1-D Heat Equation

18.303 Linear Partial Differential Equations

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Fall 2004

1 Problem 2

Find the Fourier sine and cosine series of

$$f(x) = \frac{1}{2}(1-x), \qquad 0 < x < 1.$$

a. State a theorem which proves convergence of each series. Graph the functions to which they converge.

b. Show that the Fourier sine series cannot be differentiated termwise (term-by-term). Show that the Fourier cosine series converges uniformly.

2 Problem 3

A bar with initial temperature profile $f(x) > 0$, with ends held at 0^o C, will cool as $t \to \infty$, and approach a steady-state temperature 0^oC. However, whether or not all parts of the bar start cooling initially depends on the shape of the initial temperature profile. The following example may enable you to discover the relationship.

a. Find an initial temperature profile $f(x)$, $0 \leq x \leq 1$, which is a linear combination of $\sin \pi x$ and $\sin 3\pi x$, and satisfies $\frac{df}{dx}(0) = 0 = \frac{df}{dx}(1)$, $f\left(\frac{1}{2}\right) = 4$.

b. Solve the problem

$$u_t = u_{xx}; \qquad u(0,t) = 0 = u(1,t); \qquad u(x,0) = f(x).$$

This is easy, you can just write down the solution we had in class - but make sure you know how to get it.

c. Show that for some x, $0 \leq x \leq 1$, $u_t(x,0)$ is positive and for others it is negative. How is the sign of $u_t(x,0)$ related to the shape of the initial temperature profile? How is the sign of $u_t(x,t)$, $t > 0$, related to subsequent temperature profiles? Graph the temperature profile for $t = 0, 0.2, 0.5, 1$ on the same axis (you may use Matlab).