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18.306 Advanced Partial Differential Equations with Applications
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Lecture 04 2009 09 21 MON

TOPICS: First order scalar pde.

Examples of solutions by characteristics.

Domain of influence.

Review characteristics.

Examples in detail:

1) $x \cdot u_x + y \cdot u_y = 0,$

for $y \geq 1,$ with $u(x, 1) = F(x)$

2) $x \cdot u_x + y \cdot u_y = 1 + y^2,$

for $y \geq 1,$ with $u(x, 1) = F(x)$

Domain of dependence and domain of influence. Where is the solution defined and where it is not.

Examples showing solution not unique outside domain of influence:

For case (1), with $F(x) = \exp(-x^2),$ consider (in the plane without the origin = P0)

$u_1 = \exp(-x^2/y^2)$ for $x^2 + y^2 > 0.$

$u = \exp(-x^2/y^2)$ for $y \geq 0$ and $x^2 + y^2 > 0.$

$= \exp(-3x^2/y^2)$ for $y \leq 0$ and $x^2 + y^2 > 0.$

Both u_1 and u_2 are smooth and solve the equation and given data, but they are not equal outside $y \geq 0$ and $x^2 + y^2 > 0.$ Can construct infinitely many such u 's.