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18.306 Advanced Partial Differential Equations with Applications
Fall 2009

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Lecture 17 2009 11 04 WED

TOPICS: Eikonal. Focusing and caustics. Description of the caustic.
Breakdown of approximation. Derivation of amplitude equation.

Characteristics for $H(u, p, q, x, y) = 0$; crossings and multiple values.

Example: Eikonal.

- Focusing of convex wave-fronts and caustics.
- Caustics as edge of the multiple-values region.
- Caustics as envelope of the rays.
- Caustics as the locus of the centers of curvature of the wave front.
- Typical form for caustic. Cusp at location of the first ray to focus.

Multiple-values not a problem: can have multiple waves at any given place.

However: as wavefronts approach the caustic, the expansion breaks down.

Wavelength no longer shorter than all other length scales: wave front develops large curvature as it approaches caustic.

Hence: need another approximation near caustic.

Derivation of equation for amplitude A .

- Conservation of energy (A^2) and blow up at caustics.

- Energy moves along rays at speed c .

- Characteristic form of the equation for the evolution of the amplitude.