Pak
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end of proof: And then you change edges along the path and check that it works. Check Bollobás's book.

Reminder: $\eta(G)$ = min colors to edge-color G
Vizing's Thm: $K \leq \eta(G) \leq K+1$ where K = max deg

Thm G bipartite $\Rightarrow \eta(G) = K$

But first
Hall's Politically Incorrect Theorem
(will be used in proof)

Pf: $G = H_1 \sqcup H_2$
Let $A_1 \subset H_1$ = # vert w/ deg K
$A_2 \subset H_2$ = "
Know $\exists$ matching from A_1 to H_2, as satisfies Hall (else higher average deg Σ), pull it out + color those K, new A_2' has matching to new H_1, color those K, inductively color the rest ✓

Thm ~~[illegible]~~ (Heawood 1890)
Let G have Eulerian characteristic $K \leq 1$.
Then $\chi(G) \leq \dfrac{7 + \sqrt{49-24K}}{2} = h(K)$

G has m edges, n vertices, f faces

$$n - m + f = K(G) = 2 - 2g \qquad g \text{ genus}, \; K \text{ Euler characteristic}$$

"Pf": Let G be minimal ~~counterexample~~, graph s.t. $\chi(G) = r$. Then min deg of $G \ge r-1$. Need: If G has $n \ge h(K)+1$ vertices then min deg $\le h(K-1)$

$n \ge h(K)+1 \Rightarrow m \le 3n - 3K$ (sim to $e \le 3n-6$)

$\Rightarrow$ min deg $\le \frac{2m}{n} = 6 - 6K/n \le 6 - \frac{6K}{h(K)+1}$ cheat!

Suppose min deg $\ge h$

$$6 - \frac{6K}{h+1} \ge h \Rightarrow h^2 - 5h + 6(K-1) \le 0$$

$$\Rightarrow h \le \tfrac{1}{2}\left(5 + \sqrt{49 - 24K}\right) \;\lightning$$

<u>If</u> $-6K$ is positive, then okay