

## 18.330 :: Homework 2 :: Spring 2012 :: Due March 1

1. (1 pt) Let  $\alpha$  and  $\beta$  be real numbers. If a quantity  $f(h)$  is  $O(h^\alpha)$  as  $h \rightarrow 0$ , show that

$$h^\beta f(h) = O(h^{\alpha+\beta}).$$

2. (2 pts) Consider the “midpoint rule”,

$$\int_0^1 f(x) dx \simeq h \sum_{j=0}^{N-1} f\left(\left(j + \frac{1}{2}\right)h\right), \quad h = 1/N.$$

Draw a picture to explain the “midpoint” interpretation. How accurate is the method, i.e., what is the order of accuracy? Justify your answer.

3. (3 pts) Consider the function  $f(x) = \frac{x}{1+x^4}$  over the interval  $[-1, 2]$ . To integrate this function numerically, write a script to implement the first-order rectangle rule and the second-order trapezoidal rule. (Now is a good time to learn about vector operations and the “sum” command in Matlab, if you do not already know them.)
- How do you write the quadrature formulas to deal with the fact that the interval is  $[-1, 2]$  instead of  $[0, 1]$ ? In particular, how you relate  $h$  to  $N$ , and what is your choice of grid points  $x_j$ ?
  - What is the value of  $\int_{-1}^2 f(x) dx$ , to 10 decimal places?
  - Do a log-log plot of the absolute value of the truncation error of each method as a function of  $h$ . (Choose an adequate range of  $h$  that spans several orders of magnitude.) Explain how you can spot the order of convergence of each method from your plot.
4. (1 pt) Repeat the convergence plots as in question 3, but with the function  $g(x) = (x+1)^3(x-2)^2$  on  $[-1, 2]$ . What order of accuracy do you now observe for each method? (Bonus, 1pt: explain your observation.)
5. (3 pts) Consider the same function  $f(x) = \frac{x}{1+x^4}$  on  $[-1, 2]$ . To differentiate this function numerically, write a script to implement the first-order forward difference and the second-order centered difference. We are interested in computing the derivative not just at one point, but at every point of a fine grid with spacing  $h$  (except at the two endpoints). For simplicity, you may want to use the same grid convention as in the previous questions.
- What is the value of  $f'(1)$ , to 10 decimal places?
  - Do a log-log plot of the truncation error of each method as a function of  $h$ . Measure the truncation error as the maximum over all the gridpoints (except the 2 endpoints) of the absolute value of the difference between the numerical derivative and the “exact” derivative. A very accurate numerical derivative is a good substitute for “exact”. Explain how you can spot the order of convergence of each method from your plot.

(You are free to use any method when asked about numerical values, including analytical/symbolic, but please mention how you obtained your answer.)

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