18.338J/16.394J: The Mathematics of Infinite Random Matrices

Professor Alan Edelman

Handout #1, Thursday, September 9, 2004 — Course Outline

Summary. This is a course on the mathematics and applications of infinite random matrices. Our aim is to touch upon various branches of the study of infinite random matrices—a consequence is that we will end up lingering on some areas longer than others. Our hope is that this course will confer:

- some familiarity with several of the main thrusts of work in infinite random matrices—sufficient to give you some context for formulating and seeking known solutions to applications in engineering and physics;
- sufficient background and facility to let you read current research publications in the area of infinite random matrix theory;
- a set of tools, both analytical and computational, for the analysis of new random matrices that arise in new problems you may encounter.

Lecturer. Alan Edelman

Content. The goal is for the course is, paradoxically, to be broad *as well as* deep. Our plan (unlikely to survive contact with the trials and tribulations of an actual semester here at MIT) is to touch upon the following broad areas while attempting to uncover deep insights into the underlying mechanisms that unify these areas. This is a tentative list of topics that *might* be covered in the course; We will select material adaptively based on our background, interests, and rate of progress. If you are interested in some other topics, please let us know and we'd be happy to accomodate your interests.

Combinatorial aspects. Using combinatorial techniques to derive the limiting distribution of the three classical random matrix ensembles. Path counting and random matrix theory. Generalizations to counting paths on torii.

Stieljtes transform based methods. The Marčenko-Pastur theorem. Other generalizations. Silverstein's sample covariance matrix. Convergence issues.

Free probability. The concept of freeness. Free cumulants and non-crossing partitions. The R and S transform. Fluctuations and Second Order Freeness. Combinatorial interpretations.

Equilibrium Measure. The Hermite, Laguerre and Jacobi orthogonal polynomials. Interpretation of the limiting distribution as the equilibrium measure of (univariate) orthogonal polynomials. Applications to physics.

Fredholm Determinants. Tracy-Widom Distribution. Eigenvalue spacings and the Riemann Hypothesis.

Jack Polynomials. Multivariate orthogonal polynomials. Combinatorial aspects. Connections to random matrices.

Applications. Wireless Communications, Statistical Physics.

Prerequisites. We assume that the reader has had an undergraduate course in Linear Algebra (18.06) or its equivalent and some exposure to probability (6.041 or 6.042 are more than sufficient). Knowledge of combinatorial theory is a bonus.

Requirements. Problem sets, mid-term project and final project.

Homework. Homework assignments will be handed out bi-weekly. They will mainly consist of `MATLAB` based explorations of the material covered in class. You will not be needed to turn them in although being able to do them will greatly help your understanding of the material.

Mid-Term Project. You will be asked to read a paper on a topic of interest to you that involves random matrix theory and present it via some mixture of the following perspectives:

- Write a description *of greater clarity* than the original publication, or
- Devise an improved solution to the problem under consideration, and write up your improvement (with appropriate discussion of the original solution).
- Implement the result in `MATLAB` in order to study its performance in practice when the random matrices are finite. Considerations include choice of random matrix result, design of good tests, interpretation of results, and design and analysis of heuristics for improving performance in practice.

Semester project. The semester project can be an extension of the mid-term project if it sustains your interest. Otherwise, you will be asked to come up with some insights into a random matrix problem that is of interest to you.

Ideally, the topic you choose will be motivated by a computational problem you need to solve in your research. If the project is large, it may be tackled by a group.

Otherwise, feel free to ask us for suggestions and ideas. Our goal is that you are able to get your hands wet trying to solve a problem while using computational and analytical tools that you've learned about.

In the past, a student's whole PhD thesis came out of such an exploration. While that might be the exception and not the norm we hope we can instil a sense of adventure in you to tackle an interesting random matrix problem and to, hopefully, get some useful results out of it.

In this spirit, and since this is an advanced graduate class on a very active research area, the grading will be based on your participation in the class.

We encourage you to interact with the teaching staff. We are all very enthusiastic about random matrices and will be happy to discuss anything of interest to you (though we can only guarantee the veracity of our remark if it involves random matrices).

Textbooks. There are no textbooks covering a majority portion of the material we will be studying in this course. We will be giving out course readers during the semester to help you study the material before the lectures. There will be reserch papers handed out in class and posted on the website.

Guest Speakers.

We will have guest speakers that will help give us their own perspective on their research involving random matrices. While these speakers will appear as part of the Applied Math Colloquium series, attendance is strongly encouraged. The confirmed dates for two of the speakers are:

- Dr. Anna Scaglione. Monday, October 25, 2004, 4:15 pm in 2-105
- Dr. Roland Speicher. Monday, November 18, 2004, 4:15 pm in 2-105