MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.111J/18.435J/ESD.79
Quantum Computation
Fall 2004

Problem Set 8
Due: Tuesday, November 16 (in class)

Problem 1. Write out

$$U = e^{-i\pi(\sigma_X \otimes I)/4} e^{-i\pi(\sigma_Z \otimes \sigma_Z)/4} e^{-i\pi(\sigma_Y \otimes I)/4}$$

as a 4 by 4 matrix . How does it differ from a CNOT gate? How can you supplement the above circuit with single-qubit rotations to construct an exact CNOT (up to an overall phase)?

Problem 2. (Decoupling). Show that

$$U = e^{-i\pi(\sigma_X \otimes I)/2} e^{-iHt} e^{-i\pi(\sigma_X \otimes I)/2} e^{-iHt}$$

only acts on the second qubit, where

$$H = \frac{1}{2}(-\omega_1 \sigma_Z \otimes I - \omega_2 I \otimes \sigma_Z + \gamma \sigma_Z \otimes \sigma_Z) .$$

Hint 1. If $[H_1, H_2] = 0$, then $\exp(-i(H_1 + H_2)t) = \exp(-iH_1 t)\exp(-iH_2 t)$.
Hint 2. You may find it useful to first find the following operations
$(\sigma_X \otimes I)\exp(-i\omega t(\sigma_Z \otimes I)/2)(\sigma_X \otimes I)$, $(\sigma_X \otimes I)\exp(-i\omega t(I \otimes \sigma_Z)/2)(\sigma_X \otimes I)$, and $(\sigma_X \otimes I)\exp(-i\omega t(\sigma_Z \otimes \sigma_Z)/2)(\sigma_X \otimes I)$.

Problem 3. Create a decoupling scheme for a 3-qubit molecule with the following Hamiltonian:

$$H = \frac{1}{2}(-\omega_A \sigma_Z^A - \omega_B \sigma_Z^B - \omega_C \sigma_Z^C + \Gamma_{AB} \sigma_Z^A \otimes \sigma_Z^B + \Gamma_{BC} \sigma_Z^B \otimes \sigma_Z^C + \Gamma_{AC} \sigma_Z^A \otimes \sigma_Z^C) .$$

In other words, find the decoupling operators U_k such that

$$U = \prod_{k=1}^{n} U_k e^{-iHt}$$

operates only on individual qubits. (You might come up with 3 or 4 decoupling operators depending on what you eventually get.)