MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.111J/18.435J/ESD.79
Quantum Computation
Fall 2004

Problem Set 9
Due: Tuesday, November 23 (in class)

Problem 1. For the position operator x and the momentum operator p (corresponding to $-i\hbar\dfrac{\partial}{\partial x}$), show that $[x, p] = i\hbar$.

Problem 2. Verify the following equalities:
(a) $[a, a^\dagger] = 1$
(b) $[a, a^\dagger a] = a$
(c) $[a^\dagger, a^\dagger a] = -a^\dagger$
(d) $H = \hbar\omega(a^\dagger a + 1/2)$
where

$$a = \frac{1}{\sqrt{2\hbar\omega}}(\omega x + ip)$$

and

$$H = \frac{1}{2}(p^2 + \omega^2 x^2).$$

Problem 3. For $\hbar = \omega = 1$, verify $a|0\rangle = 0$, where

$$|0\rangle = \int e^{-x^2/2}|x\rangle\, dx.$$

Problem 4. Verify

$$a|n\rangle = \sqrt{n}|n-1\rangle$$

and

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

where

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle.$$

Also, show that $|n\rangle$ is an eigenstate of H with energy (eigenvalue) $\hbar\omega(n+1/2)$.

Problem 5. Show that $[C, D] = [C, H] = [D, H] = 0$, where

$$C = \omega(a^\dagger a + \sigma_Z/2),$$
$$D = \kappa(a^\dagger \sigma_- + a\sigma_+) - (\Delta\omega/2)\sigma_Z,$$

and

$$H = \hbar(C + D + \omega/2).$$

(Remember that $\sigma_\pm = (\sigma_X \pm i\sigma_Y)/2$, or equivalently $\sigma_+ = |0\rangle\langle 1|$ and $\sigma_- = |1\rangle\langle 0|$.)

Problem 6. Find the cluster state for a square. In other words, find the state $|\psi\rangle$ such that

$$\sigma_X^A \sigma_Z^B \sigma_Z^D |\psi\rangle = \sigma_X^B \sigma_Z^C \sigma_Z^A |\psi\rangle = \sigma_X^C \sigma_Z^D \sigma_Z^B |\psi\rangle = \sigma_X^D \sigma_Z^A \sigma_Z^C |\psi\rangle = |\psi\rangle$$

where the super indices A, B, C, and D are associated with the vertices of the square.