MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quantum Computation
Fall 2004

Problem Set 7
Due: Tuesday, November 9 (in class)

Problem 1. Verify Trotter formula:

$$\left(e^{-iBt/n}e^{-iAt/n}\right)^n = e^{-i(A+B)t} + O(1/n)$$

where A and B are Hermitian operators.

Problem 2. Using the fact that if we can perform Hermitian operators A and B, then we can perform $\pm i[A,B]$ as well, show how we can get operators $\sigma_i \otimes \sigma_j$ from the set of operators $\sigma_Z \otimes \sigma_Z$, $\sigma_i \otimes I$, and $I \otimes \sigma_j$, where $i,j \in \{X,Y,Z\}$.

Problem 3. Use the same trick to show how to produce an arbitrary operator

$$\sigma_{i_1} \otimes \sigma_{i_2} \otimes \cdots \otimes \sigma_{i_n}.$$

Problem 4.
(a) Construct a CNOT gate using only $e^{-i\phi\sigma_Z\otimes\sigma_Z}$ and $e^{-i\theta\sigma}$ gates.
(b) What is the smallest number of one-qubit gates and the two-qubit gate $\sigma_Z \otimes \sigma_Z$ needed for the construction of a CNOT gate.

Problem 5. Verify

(a) $\cos(\omega t)\sigma_X - \sin(\omega t)\sigma_Y = e^{i\omega t}\sigma_+ + e^{-i\omega t}\sigma_-$
(b) $e^{\pm i\omega t}\sigma_\pm e^{i\omega t\sigma_Z/2} = e^{i\omega t\sigma_Z/2}\sigma_\pm$

where $\sigma_\pm = (\sigma_X \pm i\sigma_Y)/2$.