

Before remark I.2.4, the in the exact sequence 3 and 1 need to be interchanged.

In Exercise before Lemma II.5.4, u'_1 should be u_1 .

In Corollary III.1.5, you probably need that R is a field.

Proof of Proposition III.1.6. The proof can be slightly simplified. You need to show that any ideal I in $A \otimes B$ is generated by its intersection with B . By taking quotient of B by this intersection, you may assume that this intersection is zero, and you need to show that $I = 0$. This assumption makes proof a bit shorter.

2-nd paragraph of proof of III.2.10. "Next, left multiplication by a nonzero element $z \in D_-$ defines a bijection..." (multiplication by i maps D_+ to D_+ , so it's incorrect)

Corollary. III.6.7, line 3: $1 \otimes y$ should be $y \otimes 1$

p.33, line 5: free bundle \longrightarrow free module

Before Ex. III.11.1: x_{x_n} should be f_{x_n}

Example III.11.7, line 2: F should be f

p.40, line 8: $e_{ij}e_{kl} = \delta_{kj}e_{il}$

p.40, line 10: r^3 should be r^4

Second equation of III.11.9: $\sum_{i,\alpha} x_{ii\alpha} = 1$

Proof of III.12.1, it would be good to explain in (i)=~~i~~(iii) why A is projective (A_S is projective, and tensoring with S maps non-exact to non-exact since S is faithfully flat)

In Theorem III.13.1, you want to assume irreducibility of X in (iv), not only in (iii).

Theorem III.14.3, line 3: $< D$ should be $\leq D$

After (V.2.3) (p.62), "do show" should be "to show"

In formula (V.4.2), σs should be σN .

In Prop. V.6.1(ii), need to assume that V is faithful

Lemma V.6.3: delete "is D".

Theorem V.7.1: delete "for example a prime algebra" (this is incorrect: L. Small explained that there are prime algebras with nonzero nil ideals).

p.71, second paragraph from the bottom: c depends on more than two variables, so it is not good to write $c(u, v)$, $u, v \in I$. Same remark in the proof of Prop. V.15.2 (p. 83).

Prop V.5.1: It seems that the assumption that k is infinite is not needed (if you restate the proposition appropriately; over \mathbb{F}_2 you of course have $x^2 - x = 0$, but $x \neq 0$). Namely, over any field, if A satisfies any polynomial identity of degree d , then it satisfies a multilinear identity of that degree. Indeed, if f is an identity of degree d , and if its degree in some variable x_1 is $m > 1$, then we consider $g(y, z, x_2, \dots, x_n) = f(y + z, x_2, \dots, x_n) - f(y, x_2, \dots, x_n) - f(z, x_2, \dots, x_n)$, which is nonzero and has degrees in y and z less than m . Continuing this way, we can assume that f has degree 1 with respect to all variables. If f is not homogeneous in x_1 (i.e. has terms not containing x_1) then we can completely eliminate x_1 by the above procedure. Thus f can be reduced to a multilinear polynomial, as desired.

Thus, Corollary V.5.3 does not need the assumption that k is infinite, either.

Corollary V.7.4. One can reformulate the proof as follows: we will prove that this property holds for $A[t]$. This implies the property for A , since any irreducible representation of $A[t]$ is irreducible when restricted to A (t being central must act by a scalar, so the algebra generated by the image of $A[t]$ in $Mat_n(K)$ is the same as that generated by A). Theorem V.8.2. Seems that proof has a gap: you use

Amitsur's theorem (in fact, Corollary V.7.4 that relies on it), but one needs to know that A has no nonzero nil ideal, which is not true for a general prime algebra. So one needs to prove separately a theorem that a prime PI algebra has no nonzero nil ideals.

Def. V.8.6: $\rho' : A \rightarrow \text{Mat}_n(K')$.

Theorem V.8.7: it seems that a much simpler proof not using Posner's theorem should work. We replace A with A/P , where P is the kernel, so we can assume that the kernel is zero. Then A has a faithful finite dimensional irreducible representation, so by Kaplansky theorem, it is a central simple algebra. Then use the second paragraph of the proof. So Lemma V.8.5 seems unnecessary.

Proof of Theorem V.9.2, line 7: Notice that $s \rightarrow$ should read: "Notice that $N \dots$ "

Lemma V.9.9, line 1 of proof: maximal ideal \rightarrow maximal 2-sided ideal. Also perhaps it's good to explain why A/M is central simple by Kaplansky Theorem. (why does A/M have a faithful simple module?) For example, one can explain that a simple PI algebra is central simple. Also, $\gamma_i = c_i$ in this proof.

Theorem V.13.4. The fact that an element in $\text{End}(V^{\otimes n})$ commuting with every permutation is in $\text{Sym}^n(\text{End}(V))$ is indeed obvious, and needs no proof (so the following half page can be deleted). 8 lines below formula (V.15.1): is $A[\gamma^{-1}]$ equal to A' used below?

The exercise 2 lines below VI.1.3 (p.85) is unclear. Should r be an integer?

end of proof of Prop. VI.1.10, p.87. Replace s' by $s' + 1$ in the last two expressions, and $> s' + 1$ by $\geq s' + 1$.

Lemma VI.2.4: you need to assume that $\frac{1}{2} < q < 1$, and replace $2+q$ with $1+2q$. Also replace $q+2$ in the proof by $1+2q$.

Proof of Lemma VI.4.2, line 5: a_{p+q} should be a_{p+q-1} .

Theorem VI.4.11, proof, 4th paragraph: $d - p + q + 2 \rightarrow d - p - q + 2$.

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