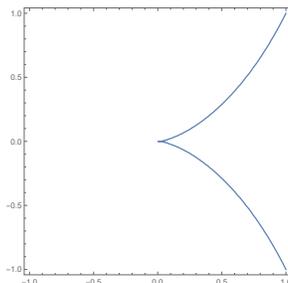


Comprehension questions

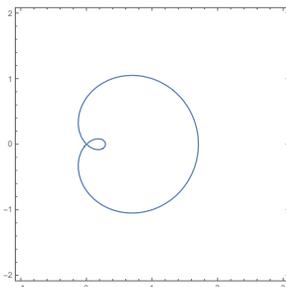
PROBLEM 23.1. *The cissoid of Diocles is the curve*

$$(x^2 + y^2)x = 2y^2$$



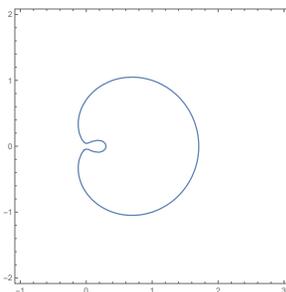
Does this have a node at $(0,0)$, and if so what kind of node? (I want an algebraic proof, not just visual inspection).

PROBLEM 23.2. *The Limaçons of Pascal are the curves $(x^2 + y^2 - x)^2 - b(x^2 + y^2) = 0$, where $b > 0$ is a parameter. Here's a sample picture, $b = \frac{1}{2}$:*

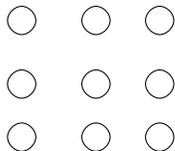


For what values of b is $(0,0)$ a node? And what kind of node? (I want an algebraic proof, not just visual inspection).

PROBLEM 23.3. *Continuing from the previous problem, here's a picture of the curve $(x^2 + y^2 - x)^2 - \frac{1}{2}(x^2 + y^2) = \epsilon$, for some small nonzero ϵ . What's the sign of ϵ ? (This should be done without computer or calculator assistance.)*



PROBLEM 23.4. Find an algebraic curve of degree 6, which qualitatively looks like this (9 loops in the plane, none of them nested inside each other):



To solve this, it's enough to include your equation and a computer-generated plot.

PROBLEM 23.5. Let $p(x)$ be a polynomial in one variable, and x_0 a number where $p(x_0) = 0$, $p'(x_0) = 0$, $p''(x_0) \neq 0$. Show that then, $p(x) - y^2 = 0$ has a node at $(x_0, y_0 = 0)$. Depending on $p''(x_0)$, what kind of node does one get?

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18.900 Geometry and Topology in the Plane
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