Comprehension questions

Problem 16.1. Take an immersed loop, and modify it by inserting a curly piece somewhere, as in the picture below. How does that affect the rotation number?

\[
\begin{array}{c}
\text{Original loop} \\
\text{Modified loop}
\end{array}
\]

Problem 16.2. Compute the rotation number of the immersed loop drawn below, in two ways: by counting tangencies; and by applying Whitney’s formula.

\[
\text{Immersed loop}
\]

Problem 16.3. Consider closed immersed curves which have exactly two simple self-intersection points. What are all possible rotation numbers? Draw examples for the rotation numbers that can occur.

Problem 16.4. The picture below looks like a deformation during which the rotation number changes by 1, contradicting the theorem from the lecture. What’s wrong?

\[
\text{Deformation}
\]

If you prefer to think in terms of formulae, you could use \( c_s(t) = (t + (1 + s)\cos(t), s\sin(t)) \), in the region \( 0 \leq t \leq \pi \) (which is not itself a loop, but could be part of a loop). However, the insight is independent of any specific formula.

Problem 16.5. What is the rotation number of the anticlockwise circle with \( k \geq 0 \) outwards curls added to it, like this: