Comprehension questions

Both the comprehension questions and the further ones are designed to be done using only the material we've explained so far (we will learn more about Betti numbers in the next lecture, but I don't want you to make use of those results yet).

PROBLEM 29.1. *Which is a planar complex, and which isn't? (As in the lecture, all points that are part of the complex appear as fat dots in these pictures, and all triangles that are part of the complex are shaded.)*
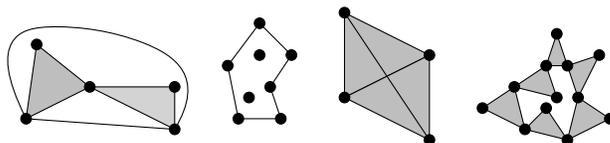
PROBLEM 29.2. *Here are the boundary operators, draw the complex:*

$$D_1 = \begin{pmatrix} -1 & -1 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}.$$

PROBLEM 29.3. *Which integers can appear as the Euler characteristic of a planar complex? Give examples (for any value that can appear) or explanations (for any that can't).*

PROBLEM 29.4. *Compute the Betti numbers of a triangle (by this, we mean a triangle with the interior filled in: $n_0 = 3$, $n_1 = 3$, $n_2 = 1$).*

18.900 Geometry and Topology in the Plane
Spring 2023