Comprehension questions

PROBLEM 8.1. *Draw all possible 7-bounce periodic trajectories for billiards in a circle (up to rotations of the circle). You do not have to justify your answer.*

PROBLEM 8.2. *Write a computer program which finds, at least approximately, periodic billiards trajectories in the ellipse $x^2/2 + y^2 = 1$, by looking for long polygonal loops with vertices on the ellipse. Draw a 5-bounce trajectory that the program found, and write down its length.*

The problem below uses the phase space for an ellipse (which is a shape with no corners). This is not a problem: you just fix an arbitrary starting point on the boundary, and then s measures the arclength distance going anticlockwise from that point, so phase space in (s, θ) coordinates is a single rectangle

$$\Omega = [0, \text{arclength of the ellipse}) \times (0, \pi).$$

PROBLEM 8.3. *Take the trajectory from* (8.6)*, and draw an approximate picture of it in the phase space of an ellipse. In your picture, include a bunch of bounces in the future, enough to suggest the behaviour for large numbers of bounces.*

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