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PAIGE BRIGHT: Hi. My name is Paige Bright, and I'm a junior studying mathematics at MIT, and I'll be your guest host for today's episode of *Chalk Radio*. For this episode, I sat down with Professor Haynes Miller.

HAYNES I'm a professor, now emeritus, of mathematics at MIT. And I got here in 1986, a long time ago. I've taught a lot of
MILLER: subjects to undergraduates, especially 18.03, Differential Equations. And I hope we have a chance to talk about that. That's who I am.

PAIGE BRIGHT: I first met with Haynes when he was my instructor for a special First-Year Seminar at MIT. I'll let Haynes tell you more about these seminars he facilitated.

HAYNES MIT has a wonderful series of classes called First-Year Advising Seminars. These are small seminars, maybe eight
MILLER: students. The seminar that you and I were in, Paige, was unusual because this was the first Fall of the pandemic, so it was entirely remote.

Very interesting. We had many time zones represented. One student was calling in from Vietnam, another from Kuwait, another from Nepal, and places in the US. And it was the most successful seminar that I've run because it was a chance for students to interact on a personal level with their peers, even though they were halfway across the world.

PAIGE BRIGHT: One of the most important aspects of these First-Year Advising Seminars is its focus on communication and mathematics. The seminars start by asking a simple question that hooks in mathematicians of any background like, how do you define symmetry, or what is a number? And in the end, it covers complex mathematical topics like moduli spaces and surreal numbers. Part of what makes Haynes's classes so compelling is his focus on the ways we communicate as mathematicians.

HAYNES In a way, I envy my engineering colleagues because they produce artifacts. They produce actual physical things.
MILLER: And my lab science colleagues do the same thing. They interact in complicated ways with the world.

Mathematicians, all we have is communication. All we have is the writing that we do. We publish and we try to explain our ideas, and there isn't any physical object that we produce. So communicating is the whole game in mathematics in a way.

PAIGE BRIGHT: Given that communicating math is so integral in Haynes's teaching and research, I was curious how he thinks about it. What's his approach?

HAYNES Whenever you're writing or communicating, the main thing is to keep your audience in mind. Who are you talking
MILLER: to? Where are they? What is their background interest? Why are they there? What is their background knowledge? And when you're teaching, that's paramount importance.

So when I was teaching Differential Equations, 18.03, I always had in mind that I was teaching mainly engineers. And there were some science majors and some math majors in my audience, but for them, this was often as close to an engineering course as they were going to see here at MIT. So I tried very hard to use engineering language.

In fact, one of the things that I would do was to invite an engineer into the classroom one day during the semester, and we would talk the talk with each other. We'd use the language that we'd been developing in the class. And it became clear to the students that this was real. This was the language that engineers used. Of course, the engineers always knew much more about the kind of systems and signals language than I was talking about, so I formed a kind of translating device between the students who knew little and the engineers who knew a great deal.

PAIGE BRIGHT: So let's talk for a minute about this class that Haynes is referring to, Differential Equations or 18.03. The study of differential equations is the study of how small changes in a function or derivatives impact the function itself. For instance, how does heating a metallic rod at one point affect the temperature of the rod everywhere?

In a class like 18.03, one hopes to systematically understand these types of problems. But Haynes has a bit of a different approach to teaching differential equations, one that changes how students think about functions as a whole.

HAYNES
MILLER: Students come into a Differential Equations course, and they've seen calculus, and they're getting used to the idea of a function. But still for many of them, a function, it's a complicated thing. It has many parts. There's many input values, and each one has an output value. It's a complicated thing.

And what happens in the course of a Differential Equations course is you come to think of a function as a single thing. It's a *composite* thing. It has smaller parts, but you think of it as a single object. That process of collecting a group of things and coalescing them into a single object, that's a fundamental mathematical operation that you can see in action already very clearly in a differential equations course.

And it's one of the main skills that I hope students come away with from taking a course like this. They think of a function as a single object that you can manipulate. You can compare two of them because one is the derivative of another or they enter into a differential equation together.

PAIGE BRIGHT: Before we get too deep into the math here, I want to take a minute to focus on Haynes's innovative approach to teaching. I asked him about how he makes math not just comprehensible, but also engaging for students.

HAYNES
MILLER: We human beings are-- most of us are equipped-- we're all equipped with a visual cortex. We're all equipped with an extremely highly developed, elaborate, and sophisticated visual system. Even if we're blind, that system is still at work in our brain. It's still available in our brain.

And what does it do? An infant looks around in the world and sees a million pixels, sees a million spots of color. But one of the first operations that an infant performs in growing up is learning the notion of an object, putting together a collection of pixels and seeing that they constitute an object that has its own identity.

So our visual cortex is really well suited to doing exactly this operation of assembling things and seeing them as a single unit. It's a special feature that we have in our brain.

So one of the things that I did to help students make this step in the Differential Equations course was to create a series of computer manipulatives, I call them. Programs. You can find them on the web at mathlets.org. And they display functions as units, and they show how the functions will change when you change parameters. They show you the relationship between functions that participate in a differential equation. And I think that for many students, seeing the graph, visually seeing the graph of a function is a huge asset in making them see this as a single object.

PAIGE BRIGHT: In keeping with his work helping students visualize differential equations, Haynes noticed that there was a big part of the academic experience that was missing a mathematical element beyond visualization: lab work.

HAYNES MIT has a general institute requirement. All undergraduates are required to take a course that has a designation
MILLER: as a lab course. Historically, mathematics didn't have a lab course. The wisdom was that, how could you have a laboratory in mathematics? There was no artifact.

But I felt that mathematics is a science. We employ the scientific method. We collect data, perhaps by doing examples, perhaps by computer. We form hypotheses. Maybe we call them conjectures. And we try to get evidence for them. Our evidence typically is called a proof. There's not a bench with lab equipment on it that people are working on together, but there is the blackboard or the whiteboard or the tablet these days of Zoom where people are working in parallel.

PAIGE BRIGHT: So together with his colleague in the math department, Professor Michael Artin, Haynes pulled together a series of small, open-ended research projects that undergraduate students could work on to fulfill their lab requirement in a math-focused environment. This led to the creation of a class entitled the Project Laboratory in Mathematics.

HAYNES It's a very high communication kind of course, but it also gives a pretty honest introduction to the research
MILLER: process in mathematics. It's something that's been useful for students who come to it never having engaged in any kind of research before. It opens their eyes to the notion that not everything is a textbook exercise. And for people who have had research experience, there also it's a great training in teamwork and in writing, in communicating.

PAIGE BRIGHT: One part of my conversation with Haynes that I really loved was his explanation of how teaching these courses led to his *own* understanding of what goes into effective math communication.

HAYNES After I got involved with these communication-intensive subjects, I became much more aware of the sort of
MILLER: stylistic issues in mathematics writing. And I have to say, I became a much more severe editor and reviewer, and worked quite hard to help the author use language correctly and be more careful about their audience, because it's very common in mathematics to begin a paragraph with a statement that comes out of the blue, because there's an image that we're struck by lightning, and we have this inspiration.

That's of course not true. There's a big background to that. There's a lot of preparation that goes into that. And writing should be the same way. You build up to a point, and then you connect. You make the next step.

PAIGE BRIGHT: Haynes has been collaborating with Susan Ruff, a lecturer in Writing Across the Curriculum, who has been working with the Mathematics Department and the Project Lab for many years now.

HAYNES Susan Ruff is very clear on the importance of what she calls "connectivity," which is to say you don't introduce a lot of new concepts simultaneously. You begin where the last sentence ended and you take the next step. And when you think of mathematics writing in that way, you really improve the comprehensibility of the writing. So I think that my work as an editor and as a reviewer of math papers has really been influenced by the communication-intensive teaching that I've been involved with here.

PAIGE BRIGHT: Haynes's clear intention to constantly be improving his teaching is truly remarkable. One way he's helped both himself and other instructors develop their teaching skills is through a program he created and hosts called the Online Seminar on Undergraduate Mathematics Education, or OLSUME.

HAYNES So about six years ago or so, I was sitting in a coffee shop with a former postdoc, Grace Lyo, from the Math Department here. And she was making the point that you research universities like Stanford where she was employed at the time and MIT where she used to be employed, the professors there think hard about teaching, but they don't talk to each other very much about their thoughts about teaching. We really don't talk too much with each other normally within a given faculty about teaching.

So we thought, well, there are these few campuses. Maybe we should have a meeting together. But it occurred to me that we have Zoom, which existed back before COVID, believe it or not, and we could do this remotely and start up a monthly seminar-- every two weeks, actually, seminar-- asking faculty to talk about the teaching initiatives that they'd engaged in in their own campus.

Sometimes it was a specific, different way of grading. Sometimes it was a different way of organizing basic courses. Sometimes it was interacting with engineers to optimize the basic courses that the Math Department was offering. And we've been running it every two weeks during the academic year ever since. It's become quite a useful resource.

I think my original thought that there was a lot of activity going on around the country, in different departments, with a lot of imagination and creativity and passion, it was true. And we have it on record now, archived and available at the OLSUME website.

PAIGE BRIGHT: On top of teaching the Differential Equations course and running the Project Lab, Haynes also runs the First-Year Seminars that we mentioned earlier. The title of the seminar / was in was titled What Is a Number? And while you might be surprised that a math major would take a class titled What Is a Number?, it's actually a rather fascinating question.

HAYNES We all think we know what numbers are. We use them every day. But very quickly, you get into some real mathematical questions that aren't buried at the end of some course. They're right on the surface. And you realize, for example, that you can't really answer that question, what is a number? It doesn't make sense in itself.

You can answer a question like, what is a number *system*? Because the whole point about numbers is how they relate to each other. So we get back to this issue about things forming complexes, things forming combinations before they really have meaning.

That was one of the most interesting and fun things about the seminar that we were in, is thinking carefully about the concept of number and the many different varieties of numbers that there are. But every time, every one of them, you see that it doesn't make sense until you think of them in a system.

Other mathematical concepts that came out very clearly was the notion that you get at something by characterizing its properties. So I guess this is the axiomatic system. You gain much better insight into what an integer is by thinking about it axiomatically.

What are the properties that you expect that number system to have? You can construct them in different ways, but every single construction of them is artificial and in a sense irrelevant, because in the end you're just using their properties. So this is a very deep mathematical perspective actually that came out quite clearly right away, without a great deal of background, and it was accessible to freshmen pretty easily.

PAIGE BRIGHT: Haynes is truly dedicated to helping people understand complex and intimidating subjects. He also cares about crafting his materials in a way that gets students thinking differently. And there's one seminar where that certainly comes to fruition, the Kan Seminar.

HAYNES
MILLER: So Dan Kan was a faculty member here. He was here when I got here. He's now deceased. He ran a graduate course, but he declined to lecture. Instead, he had students lecture. And this was a revolutionary concept I think when it began back in about 1965.

The Kan Seminar has been in action every fall ever since then, run by him first and then by other people, and then for a number of years by me. Now it's passed on to other people.

It's a literature seminar on the classic papers in this subject, algebraic topology. And it gives students-- it gives the graduate students-- and now increasingly undergraduates from MIT and from Harvard will take it. It gives them a chance to appreciate how the subject developed and to really learn the basic examples that are still motivating the subject today. It gives students a chance to talk to a faculty member regularly.

I talk with each student every week about the paper that they're going to present, and learn to read a paper in two ways. They learn to read a paper in great detail because they're going to give a talk about it. They have to understand it. And they will pick some part of the paper that they think is especially interesting and they want to tell their peers about.

But also, they're expected to read the papers that everybody else is talking about. So they also learn to read a paper very quickly and get something useful from it very efficiently.

PAIGE BRIGHT: I was curious about how he had students teach the lectures. Haynes told me a bit about his own beginnings as an instructor, and also how he gets students to present in ways that are both engaging and useful to others.

HAYNES
MILLER: I remember when I first got here, I was videotaped, as many beginning teachers are here, and I watched the video of myself teaching. And I thought, why is this guy trying to act so cool? And why doesn't he just say things like they are? And so I learned to be more direct from that experience.

Students have a tendency to assume that everybody in their audience knows everything they're going to say, and so they'll rush through it. They'll not do an adequate job in explaining what they have to say, because they think there's no point. This is just a formality of some kind.

So really persuading them that they're saying something that the audience doesn't know, and it's up to them [LAUGHS] to explain it. That's a big deal.

PAIGE BRIGHT: To close our episode today, I wanted to share some reflections from Haynes about OCW. Having personally worked on some course content for OpenCourseWare, I can say that it's an amazing experience to get to make materials that help educators *and* students alike learn from the amazing resources at MIT, regardless of where they are or what academic avenues are available to them.

HAYNES
MILLER: The MIT faculty are so incredibly lucky to have OpenCourseWare as a conduit, as a way of expressing yourselves, as a way of publishing this work that we do, that we spend a great part of our professional life working on. Other universities don't have that luxury, typically. They don't have a way to memorialize and contribute their teaching work to the world at large. It's such a great resource that we have here. One of the great things about OCW is you run the course, and then you tell OpenCourseWare to go run with it, and something *amazing* happens.

PAIGE BRIGHT: Professor Miller shares additional thoughts on the courses talked about today, such as the Kan Seminar and the Project Lab in Mathematics, on OCW. You can view these resources at ocw.mit.edu. You can also learn more about his work with the Online Seminar on Undergraduate Mathematics Education, or OLSUME at olsume.org.

Thank you so much for listening to this guest podcast. Until next time, I'm Paige Bright from MIT OpenCourseWare. MIT *Chalk Radio's* producers include Sarah Hanson, Brett Paci, and Dave Lishansky. Show notes for this episode were written by Peter Chipman. We're funded by MIT Open Learning and supporters like you.