Error Probability for Optimum Combining of M-ary PSK Signals in the Presence of Interference and Noise

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***Abstract*—An exact expression for the symbol-error probability (SEP) for coherent detection of M-ary phase-shift keying using an array of antennas with optimum combining in a Rayleigh fading environment is derived, based on the theory of orthogonal polynomials. In particular, performance analysis in the presence of multiple uncorrelated equal-power cochannel interferers and thermal noise is considered, starting from problems related to the eigenvalues distribution of complex Wishart matrices. We give an effective technique to derive the SEP involving only one integral with finite integration limits. The result is general and valid for an arbitrary number of receiving antennas and/or cochannel interferers. Based on our efficient method, new results that are useful for the design of wireless systems are obtained.**

***Index Terms*—Adaptive arrays, antenna diversity, cochannel interference, eigenvalues distribution, minimum mean-square error (MMSE) receivers, optimum combining, Wishart matrices.**

I. Introduction

The performance of wireless communication systems can be significantly improved by using arrays of antennas to reduce fading effects and suppress interference. Optimum combining (OC) has been used with adaptive arrays where the received signals are weighted and combined to maximize the output signal-to-interference-plus-noise ratio (SINR) [1]–[4]. Another technique is maximal-ratio combining (MRC), where the received signals are combined to maximize the desired signal power only [5]–[7]. The OC technique provides substantial improvement in performance over MRC when interference is present. For OC, the receiver requires the knowledge of the desired signal channel gain vector (as with MRC), and the short-term covariance matrix of the overall disturbance due to undesired interferers and thermal noise. In modern communication systems, especially in light of the on-going development of digital signal processing technology, the choice of OC is evident due to its much more efficient usage of the radio spectrum. It should, however, be emphasized that the analysis of systems with OC is more difficult than those with MRC and that the performance evaluation of the former is even more complicated if fading is taken into account for interfering and the desired signals.

Closed-form expressions for the bit-error probability (BEP) have been derived in [1] and [2] for the single interferer case under the assumption of Rayleigh fading for the desired signal. BEP expressions with Rayleigh fading of the desired signal and a single interferer are given in [8] and [9], where the results in [8] require numerical integration, and the results in [9] involve the evaluation of the Gaussian probability integral function.

With multiple interferers of arbitrary power, Monte Carlo simulation has been used to determine the BEP [2]. To avoid Monte Carlo simulation, approximations have been presented in [10] and [11] for the case of equal-power interferers. However, the approximation of [10] still requires Monte Carlo simulation to obtain mean eigenvalues (a table is provided in [10] for some cases), and the approximation of [11] has been proposed when the number of interferers is less than the number of antenna elements. The BEP expression is derived in [12] and [13], but the results are limited to the case of binary phase-shift keying (PSK) modulation, equal-power interferers, and no thermal noise.

In [14] and [15], upper bounds on the BEP of OC were derived, given the average power of the interferers. Unfortunately, these bounds are generally not tight. Recently, moving from the approach presented in [3], tighter bounds have been derived in [4], [16], and [17], in the context of multiple-input multiple-output (MIMO) systems [18]–[20]. It was shown in [3] and [4] that the exact symbol-error probability (SEP) can be derived starting from the eigenvalues distribution of Wishart complex matrices. However, numerical evaluation of SEP requires the evaluation of multiple integrals, with the number of integrals depending on the minimum of the number of antennas and interferers.

To alleviate this problem, we develop in this paper an efficient method to derive the SEP for coherent detection of M-ary PSK using OC in the presence of multiple uncorrelated equal-power interferers as well as thermal noise in a flat Rayleigh fading environment. Our new approach, based on a classical technique involving orthogonal systems, leads to exact solutions that require only the evaluation of a single integral with finite limits. Based on this new powerful tool, insightful results for the design of wireless systems are obtained. Our results are parameterized by signal-to-noise ratios (SNRs) and signal-to-interference ratios (SIRs). The (SNR, SIR)-plane corresponding to target SEP is investigated. We also obtain the increase in SNR required to

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cope with the presence of interference for a given target quality of service.

The paper is organized as follows. In Section II we provide the system description, including a statistical analysis of the eigenvalues of the short-term covariance matrix. The exact SEP is derived in terms of multiple integrals in Section III. In Section IV, the efficient methods are developed to derive the SEP in terms of a single integral with finite limits. Finally, in Section V, we show some numerical results and in Section VI, we present some conclusions.

II. Preliminaries

We consider OC of multiple received signals in flat fading environment with coherent demodulation, where the fading rate is assumed to be much slower than the symbol rate. Throughout the paper, $(\cdot)^T$ is the transposition operator and $(\cdot)^\dagger$ stands for conjugation and transposition. The received signal at the N_A-element array output consists of desired signal, N_I interfering signals, and thermal noise, all being mutually independent. After matched filtering and sampling at the symbol rate, the array output vector at time k can be written as

$$\mathbf{z}(k) = \sqrt{E_\mathrm{D}}\boldsymbol{c}_\mathrm{D} b_0(k) + \mathbf{z}_\mathrm{IN}(k) \tag{1}$$

with the interference plus noise term

$$\mathbf{z}_\mathrm{IN}(k) = \sqrt{E_\mathrm{I}}\sum_{j=1}^{N_I}\boldsymbol{c}_{\mathrm{I},j} b_j(k,\tau_j) + \mathbf{n}(k) \tag{2}$$

where E_D and E_I are the mean (over fading) energies of the desired and interfering signal, respectively; $\boldsymbol{c}_\mathrm{D} = [c_{\mathrm{D},1},\ldots,c_{\mathrm{D},N_\mathrm{A}}]^T$ and $\boldsymbol{c}_{\mathrm{I},j} = [c_{\mathrm{I},j,1},\ldots,c_{\mathrm{I},j,N_\mathrm{A}}]^T$ are the desired and jth interference propagation vectors, respectively; and $\mathbf{n}(k)$ represents the additive noise. We model $\boldsymbol{c}_\mathrm{D}$ and $\boldsymbol{c}_{\mathrm{I},j}$ as multivariate complex-valued Gaussian vectors having $\mathbb{E}\{\boldsymbol{c}_\mathrm{D}\} = \mathbb{E}\{\boldsymbol{c}_{\mathrm{I},j}\} = 0$ and $\mathbb{E}\left\{\boldsymbol{c}_\mathrm{D}\boldsymbol{c}_\mathrm{D}^\dagger\right\} = \mathbb{E}\left\{\boldsymbol{c}_{\mathrm{I},j}\boldsymbol{c}_{\mathrm{I},j}^\dagger\right\} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The desired signal $b_0(k)$ belongs to the element of the signal constellation that is being considered, i.e., binary PSK, quadrature PSK, etc.

The interfering data samples are denoted by $b_j(k,\tau_j)$ for $j = 1,\ldots,N_\mathrm{I}$, where the dependence of a (random) time delay τ_j is written explicitly to emphasize the asynchronicity between the desired signal and interfering users. They can be modeled as uncorrelated zero-mean random variables, and without loss of generality $b_0(k)$ and $b_j(k,\tau_j)$ for $j = 1,\ldots,N_\mathrm{I}$ are assumed to have unit variance. The statistical distribution of $b_j(k,\tau_j)$ for $j = 1,\ldots,N_\mathrm{I}$ are known to be tractable only for some pulse shapes and modulation formats [21], [22]. A common approach is to model $b_j(k,\tau_j)$ for $j = 1,\ldots,N_\mathrm{I}$ as Gaussian random variables, giving rise to an overall Gaussian residual total interference at the output of the linear combiner [2], [8]–[15].

The additive noise is modeled as a white Gaussian random vector with independent and identically distributed (i.i.d.) elements with $\mathbb{E}\{\mathbf{n}(k)\} = 0$ and $\mathbb{E}\left\{\mathbf{n}(k)\mathbf{n}^\dagger(k)\right\} = N_0\mathbf{I}$, where $N_0/2$ is the two-sided thermal noise power spectral density per antenna element. In the following, $\mathbf{R}$ denotes the short-term covariance matrix of the disturbance $\mathbf{z}_\mathrm{IN}(k)$, conditioned on all interference propagation vectors, given by

$$\mathbf{R} = \mathbb{E}_{\mathbf{n},b_j(k,\tau_j)}\left\{\mathbf{z}_\mathrm{IN}(k)\cdot\mathbf{z}_\mathrm{IN}(k)^\dagger\right\} \tag{3}$$

and $\mathbb{E}_X\{\cdot\}$ denotes expectation with respect to X.

In this paper, the decision variable $\hat{b}_0(k)$ that serves to estimate the useful symbol $b_0(k)$ is obtained by a projection of $\mathbf{z}(k)$ on the weighting vector $\mathbf{w}$, as follows:

$$\hat{b}_0(k) = \mathbf{w}^\dagger\mathbf{z}(k). \tag{4}$$

We now derive the weight vector $\mathbf{w}$ that maximizes the output SINR defined by

$$\gamma \triangleq \frac{E_\mathrm{D}\mathbb{E}_{b_0(k)}\left\{\left|\mathbf{w}^\dagger\boldsymbol{c}_\mathrm{D} b_0(k)\right|^2\right\}}{\mathbb{E}_{\mathbf{n},b_j(k,\tau_j)}\left\{\left|\mathbf{w}^\dagger\mathbf{z}_\mathrm{IN}(k)\right|^2\right\}} = \frac{E_\mathrm{D}\left|\mathbf{w}^\dagger\boldsymbol{c}_\mathrm{D}\right|^2}{\mathbf{w}^\dagger\mathbf{R}\mathbf{w}}. \tag{5}$$

Note that the mean is here taken over the "fast" processes, i.e., the thermal noise, the desired and jth interferers' symbols, and is conditioned on the vectors $\boldsymbol{c}_\mathrm{D}$ and $\boldsymbol{c}_{\mathrm{I},j}$; in the time domain, this corresponds to averaging over a time window of a few symbol periods over which the propagation vectors for all users are assumed to be constant.

In order to find the expression of the optimum weights in the above defined sense, let us first note that SINR in (5) is invariant to a scaling of $\mathbf{w}$. Then, setting the gradient of (5), with respect to $\mathbf{w}$, to zero gives

$$\boldsymbol{c}_\mathrm{D}\mathbf{w}^T\boldsymbol{c}_\mathrm{D}^*\mathbf{w}^\dagger\mathbf{R}\mathbf{w} = |\mathbf{w}^\dagger\boldsymbol{c}_\mathrm{D}|^2\mathbf{R}\mathbf{w} \tag{6}$$

that, recalling the scaling invariance, means

$$\boldsymbol{c}_\mathrm{D} \propto \mathbf{R}\mathbf{w}. \tag{7}$$

Furthermore, if $\mathbf{R}$ is nonsingular, the OC solution is explicitly obtained as

$$\mathbf{w} = \eta\mathbf{R}^{-1}\boldsymbol{c}_\mathrm{D} \tag{8}$$

with η an arbitrary constant.

The (maximum) SINR at the output of the N_A-element array with OC can, therefore, be expressed substituting (8) into (5) as [2]

$$\gamma = E_\mathrm{D}\boldsymbol{c}_\mathrm{D}^\dagger\mathbf{R}^{-1}\boldsymbol{c}_\mathrm{D} \tag{9}$$

where it is important to remark that $\mathbf{R}$, and consequently also the SINR γ, varies at the fading rate.

A. Relation Between OC and MMSE

For what follows, it is worthwhile to recognize that (7), with a proper choice of the scaling factor, also provides the minimum mean-square error (MMSE) between the estimated symbol and the transmitted one. In fact, defining the mean square error (MSE) as

$$J = \mathbb{E}\left\{\left|\hat{b}_0(k) - b_0(k)\right|^2\right\} = \left|\sqrt{E_\mathrm{D}}\mathbf{w}^\dagger\boldsymbol{c}_\mathrm{D} - 1\right|^2 + \mathbf{w}^\dagger\mathbf{R}\,\mathbf{w} \tag{10}$$

and setting the gradient of J in (10) equal to zero, it is found that the MMSE criterion weights must satisfy

$$\mathbf{Rw} = \sqrt{E_{\mathrm{D}}}\left(1 - \sqrt{E_{\mathrm{D}}}\mathbf{w}^{\dagger}\boldsymbol{c}_{\mathrm{D}}\right)\boldsymbol{c}_{\mathrm{D}} \tag{11}$$

which is in the form (7). Moreover, if $\mathbf{R}$ is nonsingular, the explicit MMSE solution is readily obtained as

$$\mathbf{w} = J_{\min}\sqrt{E_{\mathrm{D}}}\mathbf{R}^{-1}\boldsymbol{c}_{\mathrm{D}} \tag{12}$$

where the constant $J_{\min}$ is the minimum residual MSE given by

$$J_{\min} = \left(1 + E_{\mathrm{D}}\boldsymbol{c}_{\mathrm{D}}^{\dagger}\mathbf{R}^{-1}\boldsymbol{c}_{\mathrm{D}}\right)^{-1}. \tag{13}$$

Thus, we have shown that MMSE and OC receivers are equivalent and will be used interchangeably throughout the paper.

B. SINR Analysis

In order to study the performance of OC, we start from the SINR expression (9). Let $\tilde{\mathbf{R}} = \mathbf{C}_{\mathrm{I}}\mathbf{C}_{\mathrm{I}}^{\dagger}$ be an $(N_{\mathrm{A}} \times N_{\mathrm{A}})$ random matrix where $\mathbf{C}_{\mathrm{I}}$ is defined by

$$\mathbf{C}_{\mathrm{I}} \triangleq \begin{bmatrix} | & | & & | \\ \boldsymbol{c}_{\mathrm{I},1} & \boldsymbol{c}_{\mathrm{I},2} & \cdots & \boldsymbol{c}_{\mathrm{I},N_{\mathrm{I}}} \\ | & | & & | \end{bmatrix} \tag{14}$$

is an $(N_{\mathrm{A}} \times N_{\mathrm{I}})$ matrix composed of N_{I} interference propagation vectors as columns. The SINR can be written in terms of the eigenvalues $(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_{N_{\mathrm{A}}})$ of $\tilde{\mathbf{R}}$ as

$$\gamma = E_{\mathrm{D}} \sum_{i=1}^{N_{\mathrm{A}}} \frac{|u_i|^2}{E_{\mathrm{I}}\tilde{\lambda}_i + N_0} \tag{15}$$

where vector $\mathbf{u} = [u_1, \ldots, u_{N_{\mathrm{A}}}]^T$ is given by $\mathbf{u} = \mathbf{U}^{\dagger}\boldsymbol{c}_{\mathrm{D}}$ for some unitary matrix $\mathbf{U}$ arising from the diagonalization of $\mathbf{R}$ [4]. Since $\mathbf{U}$ represents a unitary transformation, the distribution of $\mathbf{u}$ is the same as that of $\boldsymbol{c}_{\mathrm{D}}$. Note that the eigenvalues vary at the fading rate. The joint probability density function (pdf) of $(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_{N_{\mathrm{A}}})$ was derived in [3] and [4] using the theory of multivariate statistics, relating to complex Wishart matrices [23]–[26]. It can be shown that the general expression for the joint pdf of the first $N_{\min} \triangleq \min\{N_{\mathrm{A}}, N_{\mathrm{I}}\}$ *unordered* eigenvalues $\tilde{\boldsymbol{\lambda}} = [\tilde{\lambda}_1, \ldots, \tilde{\lambda}_{N_{\min}}]^T$ of $\tilde{\mathbf{R}}$, valid for arbitrary N_{A} and N_{I}, is

$$\begin{aligned} &f_{\tilde{\boldsymbol{\lambda}}}(x_1, \ldots, x_{N_{\min}}) \\ &= \frac{K}{N_{\min}!}\left[\prod_{i=1}^{N_{\min}} e^{-x_i} x_i^{N_{\max}-N_{\min}}\right] \prod_{i=1}^{N_{\min}-1}\left[\prod_{j=i+1}^{N_{\min}} (x_i - x_j)^2\right] \end{aligned} \tag{16}$$

where $N_{\max} \triangleq \max\{N_{\mathrm{A}}, N_{\mathrm{I}}\}$ and K is the normalizing constant given by

$$K = \frac{\pi^{N_{\min}(N_{\min}-1)}}{\tilde{\Gamma}_{N_{\min}}(N_{\max})\tilde{\Gamma}_{N_{\min}}(N_{\min})} \tag{17}$$

with

$$\tilde{\Gamma}_{N_{\min}}(n) = \pi^{N_{\min}(N_{\min}-1)/2} \prod_{i=1}^{N_{\min}} (n-i)!\,. \tag{18}$$

The additional $N_{\mathrm{A}} - N_{\min}$ eigenvalues of $\tilde{\mathbf{R}}$ are identically equal to zero. Note that the joint pdf derived in [3] and [4] was for the case of ordered eigenvalues. Here, we consider the unordered distribution, which is a reason for the difference in a factor of $N_{\min}!$.

III. Derivation of the SEP

The SEP for OC in the presence of multiple cochannel interferers and thermal noise in a fading environment was derived in [3] and [4], using the chain rule of conditional expectation, as

$$P_e = \mathbb{E}_{\tilde{\boldsymbol{\lambda}}}\left\{\underbrace{\mathbb{E}_{\mathbf{u}|\tilde{\boldsymbol{\lambda}}}\left\{\Pr\left\{e \middle| \gamma = E_{\mathrm{D}} \sum_{i=1}^{N_{\mathrm{A}}} \frac{|u_i|^2}{E_{\mathrm{I}}\tilde{\lambda}_i + N_0}\right\}\right\}}_{P_{e|\tilde{\boldsymbol{\lambda}}}}\right\} \tag{19}$$

where $\Pr\{e|\gamma\}$ is the SEP conditioned on the random variable γ. In deriving (19), we first perform $\mathbb{E}_{\mathbf{u}|\tilde{\boldsymbol{\lambda}}}\{\cdot\}$ and average over the channel ensemble of the desired signal to obtain the conditional SEP, conditioned on the random vector $\tilde{\boldsymbol{\lambda}}$, denoted by $P_{e|\tilde{\boldsymbol{\lambda}}}$. We then perform $\mathbb{E}_{\tilde{\boldsymbol{\lambda}}}\{\cdot\}$ to average out the channel ensemble of the interfering signals.

In many cases of interest, the contribution from the cochannel interference and noise at the output of the MMSE combiner is well approximated by a Gaussian random variable [27]. This observation is also valid for OC via the equivalence of maximum SINR and MMSE and will be verified by Monte Carlo simulations in Section V. Therefore, the conditional SEP, conditioned on $\tilde{\boldsymbol{\lambda}}$, for coherent detection of M-ary PSK in the general case of N_{A} antennas and N_{I} interferers, can be written as [3], [4]

$$\begin{aligned} P_{e|\tilde{\boldsymbol{\lambda}}}(\tilde{\boldsymbol{\lambda}}) &= \frac{1}{\pi}\int_0^{\Theta} \\ &\quad \mathbb{E}_{\mathbf{u}|\tilde{\boldsymbol{\lambda}}}\left\{\exp\left(-\frac{c_{\mathrm{MPSK}}}{\sin^2\theta}E_{\mathrm{D}}\sum_{i=1}^{N_{\mathrm{A}}}\frac{|u_i|^2}{E_{\mathrm{I}}\tilde{\lambda}_i + N_0}\right)\right\} d\theta \end{aligned} \tag{20}$$

$$= \frac{1}{\pi}\int_0^{\Theta} A(\theta) \prod_{i=1}^{N_{\min}} \left[\frac{\sin^2\theta}{\sin^2\theta + c_{\mathrm{MPSK}}\frac{E_{\mathrm{D}}}{E_{\mathrm{I}}\tilde{\lambda}_i + N_0}}\right] d\theta \tag{21}$$

where $c_{\mathrm{MPSK}} = \sin^2(\pi/M)$, $\Theta = \pi(M-1)/M$, and

$$A(\theta) = \left[\frac{\sin^2\theta}{\sin^2\theta + c_{\mathrm{MPSK}}\frac{E_{\mathrm{D}}}{N_0}}\right]^{N_{\mathrm{A}}-N_{\min}} \tag{22}$$

and we have used the fact that $\mathbf{u}$ is Gaussian with i.i.d. elements in deriving (21).[1]

[1]Although this paper concerns the derivation of exact SEP, it is noted here that the integrand of (21) is Schur monotonic [28], and this fact can be used to obtain bounds on SEP.

Using (19), the unconditional SEP for OC becomes

$$P_e = \int_0^\infty \cdots \int_0^\infty P_{e|\tilde{\boldsymbol{\lambda}}}(\boldsymbol{x}) \cdot f_{\tilde{\boldsymbol{\lambda}}}(\boldsymbol{x}) dx_1 \dots dx_{N_{\min}} \tag{23}$$

where $P_{e|\tilde{\boldsymbol{\lambda}}}(\boldsymbol{x})$ is given in (21). Expression (23) is exact and valid for arbitrary numbers of antennas and interferers; however, it requires the evaluation of $N_{\min}$-fold integrals, which can be cumbersome to evaluate for large $N_{\min}$. We will show how this analytical difficulty can be avoided using the classical theory of orthogonal systems and the properties relating to the Vandermonde matrix.

IV. Efficient Evaluation of SEP for OC

We first note that the term $\prod_{i=1}^{N_{\min}-1}\left[\prod_{j=i+1}^{N_{\min}}(x_i - x_j)\right]$ in (16) can also be seen as the determinant of the Vandermonde matrix $\mathbf{V}(x_1, \dots, x_{N_{\min}})$ given by

$$\mathbf{V}(x_1, \dots, x_{N_{\min}}) \triangleq \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_{N_{\min}} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{N_{\min}-1} & x_2^{N_{\min}-1} & \cdots & x_{N_{\min}}^{N_{\min}-1} \end{pmatrix}. \tag{24}$$

Therefore, the pdf can also be written as

$$f_{\tilde{\boldsymbol{\lambda}}}(x_1, \dots, x_{N_{\min}}) = \frac{K}{N_{\min}!}\left|\mathbf{V}(x_1, \dots, x_{N_{\min}})\right|^2 \prod_{i=1}^{N_{\min}} e^{-x_i} x_i^{N_{\max}-N_{\min}}. \tag{25}$$

For what follows, it is convenient to introduce the function[2]

$$\psi(\tilde{\lambda}, \theta) \triangleq \frac{\tilde{\lambda} + \frac{N_0}{E_{\mathrm{I}}}}{\tilde{\lambda} + \frac{N_0}{E_{\mathrm{I}}} + \frac{c_{\mathrm{MPSK}}}{\sin^2\theta}\frac{E_{\mathrm{D}}}{E_{\mathrm{I}}}} \tag{26}$$

and using (25), the expression (23) becomes

$$P_e = \frac{K}{N_{\min}!}\frac{1}{\pi}\int_0^\Theta A(\theta)\int_0^\infty \cdots \int_0^\infty \left|\mathbf{V}(x_1, \dots, x_{N_{\min}})\right|^2 \times \left[\prod_{i=1}^{N_{\min}} \psi(x_i, \theta) x_i^{N_{\max}-N_{\min}} e^{-x_i} dx_i\right] d\theta. \tag{27}$$

The evaluation of (27) is difficult because the integrand does not factor and the number of integral depends on the minimum of the number of antennas and interferers. We now give an efficient method to reduce (27) to the SEP expression involving a single integral with finite integration limits. The approach is based on a classical technique commonly used in mathematical physics involving orthogonal systems [29].

Let us first consider a more general problem of evaluating

$$\mathbb{E}_{\tilde{\boldsymbol{\lambda}}}\left\{\prod_{i=1}^{N_{\min}} z(\tilde{\lambda}_i, \theta)\right\} \tag{28}$$

where $z(x, \theta)$ is a nonnegative function, and the average is with respect to the distribution of the eigenvalues given by (25). This problem can be efficiently solved by using some classical results from orthogonal polynomials, as follows.

[2]The dependence of the parameters E_{D}, E_{I}, and N_0 is suppressed to simplify the notation.

For each $\theta \in [0, \Theta]$, let $\mathcal{P}_\theta^{N_{\min}}$ be the space of all polynomials with degree less than or equal to $N_{\min} - 1$ with measure

$$d\mu_\theta(x) = z(x, \theta) x^{N_{\max}-N_{\min}} e^{-x} dx \tag{29}$$

equipped with the inner product and norm defined, respectively, by

$$\langle f, g\rangle(\theta) \triangleq \int_0^\infty f(x)g(x)z(x, \theta)x^{N_{\max}-N_{\min}}e^{-x}dx \tag{30}$$

$$\|f\|_\theta^2 \triangleq \int_0^\infty f(x)f(x)z(x, \theta)x^{N_{\max}-N_{\min}}e^{-x}dx. \tag{31}$$

If the nonnegative function $z(x, \theta)$ is such that $\mu_\theta(x)$ is increasing in at least $N_{\min}$ points of x, then the elements $1, x, x^2, \dots, x^{N_{\min}-1}$ of the Hilbert Space $\mathcal{P}_\theta^{N_{\min}}$ are linearly independent. This implies that there exists an orthogonal system $\{\phi_n(x, \theta)\}_{n=0}^{N_{\min}-1}$ with

$$\phi_n(x, \theta) = \phi_{n,0}(\theta) + \phi_{n,1}(\theta)x + \cdots + \phi_{n,n}(\theta)x^n \tag{32}$$

such that $\langle\phi_n, \phi_m\rangle(\theta) = \|\phi_n\|_\theta^2\,\delta_{n,m}$, where $\delta_{n,m}$ is the Kronecker delta function defined by

$$\delta_{n,m} \triangleq \begin{cases} 1, & n = m \\ 0, & n \neq m. \end{cases} \tag{33}$$

The orthogonal system $\{\phi_n(x, \theta)\}_{n=0}^{N_{\min}-1}$ can be obtained by a Gram–Schmidt procedure using the measure $d\mu_\theta(x)$, as shown in Appendix A. Hence, we have constructed an uncountable number of orthogonal systems, each generated by the measure $d\mu_\theta(x)$ indexed by $\theta \in [0, \Theta]$.[3]

Theorem 1:

$$\mathbb{E}_{\tilde{\boldsymbol{\lambda}}}\left\{\prod_{i=1}^{N_{\min}} z(\tilde{\lambda}_i, \theta)\right\} = K\prod_{n=0}^{N_{\min}-1}\|\phi_n\|_\theta^2 \tag{34}$$

where K is given in (17) and $\prod_{n=0}^{N_{\min}-1}\|\phi_n\|_\theta^2$ is the product norm squares of all the elements in a particular orthogonal system generated by $d\mu_\theta(x)$.

Proof:

$$\mathbb{E}_{\tilde{\boldsymbol{\lambda}}}\left\{\prod_{i=1}^{N_{\min}} z(\tilde{\lambda}_i, \theta)\right\} = \frac{K}{N_{\min}!}\int_0^\infty \cdots \int_0^\infty \left|\mathbf{V}(x_1, \dots, x_{N_{\min}})\right|^2 \times \prod_{i=1}^{N_{\min}} z(x_i, \theta) x_i^{N_{\max}-N_{\min}} e^{-x_i} dx_i. \tag{35}$$

For any given $\theta \in [0, \Theta]$, the Vandermonde matrix $\mathbf{V}(x_1, \dots, x_{N_{\min}})$ can be transformed, using the orthogonal system generated by $d\mu_\theta(x)$, into $\tilde{\mathbf{V}}(x_1, \dots, x_{N_{\min}}; \theta)$ defined by

$$\tilde{\mathbf{V}}(x_1, \dots, x_{N_{\min}}; \theta) \triangleq \begin{pmatrix} \phi_0(x_1, \theta) & \phi_0(x_2, \theta) & \cdots & \phi_0(x_{N_{\min}}, \theta) \\ \phi_1(x_1, \theta) & \phi_1(x_2, \theta) & \cdots & \phi_1(x_{N_{\min}}, \theta) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{N_{\min}-1}(x_1, \theta) & \phi_{N_{\min}-1}(x_2, \theta) & \cdots & \phi_{N_{\min}-1}(x_{N_{\min}}, \theta) \end{pmatrix} \tag{36}$$

[3]Other choices of measure lead to other orthogonal systems [29].

by means of elementary row operations. Since the determinant is invariant to such row operations [30]

$$\left|\mathbf{V}(x_1,\ldots,x_{N_{\min}})\right| = \left|\tilde{\mathbf{V}}(x_1,\ldots,x_{N_{\min}};\theta)\right|. \tag{37}$$

We now let $\mathcal{S}_{N_{\min}}$ be the set of all permutations of integers $\{0,1,\ldots,N_{\min}-1\}$ and let $\sigma \in \mathcal{S}_{N_{\min}}$ denote the particular function $\sigma : (0,1,\ldots,N_{\min}-1) \to (\sigma_1,\sigma_2,\ldots,\sigma_{N_{\min}})$ which permutes the integers $\{0,1,\ldots,N_{\min}-1\}$. The determinant can be written as

$$\left|\tilde{\mathbf{V}}(x_1,\ldots,x_{N_{\min}})\right| = \sum_{\sigma\in\mathcal{S}_{N_{\min}}} \operatorname{sgn}\{\sigma\} \prod_{i=1}^{N_{\min}} \phi_{\sigma_i}(x_i,\theta) \tag{38}$$

where

$$\operatorname{sgn}\{\sigma\} = \begin{cases} +1, & \text{for even permutation} \\ -1, & \text{for odd permutation.} \end{cases} \tag{39}$$

Substituting (37) and (38) into (35) gives

$$\begin{aligned}
&\mathbb{E}_{\tilde{\boldsymbol{\lambda}}}\left\{\prod_{i=1}^{N_{\min}} z(\tilde{\lambda}_i,\theta)\right\} \\
&= \frac{K}{N_{\min}!}\int_0^\infty\cdots\int_0^\infty \sum_{\alpha\in\mathcal{S}_{N_{\min}}}\sum_{\sigma\in\mathcal{S}_{N_{\min}}} \operatorname{sgn}\{\alpha\}\operatorname{sgn}\{\sigma\} \\
&\quad\times \prod_{i=1}^{N_{\min}} \phi_{\alpha_i}(x_i,\theta)\phi_{\sigma_i}(x_i,\theta)z(x_i,\theta)x_i^{N_{\max}-N_{\min}}e^{-x_i}dx_i \\
&= \frac{K}{N_{\min}!}\sum_{\alpha\in\mathcal{S}_{N_{\min}}}\sum_{\sigma\in\mathcal{S}_{N_{\min}}} \operatorname{sgn}\{\alpha\}\operatorname{sgn}\{\sigma\} \\
&\quad\times \prod_{i=1}^{N_{\min}} \left\langle \phi_{\alpha_i}(x_i,\theta),\phi_{\sigma_i}(x_i,\theta)\right\rangle.
\end{aligned} \tag{40}$$

Using the orthogonality property of $\{\phi_n(x,\theta)\}_{n=0}^{N_{\min}-1}$, (40) becomes

$$\begin{aligned}
\mathbb{E}_{\tilde{\boldsymbol{\lambda}}}\left\{\prod_{i=1}^{N_{\min}} z(\tilde{\lambda}_i)\right\} &= \frac{K}{N_{\min}!}\sum_{\alpha\in\mathcal{S}_{N_{\min}}}\sum_{\sigma\in\mathcal{S}_{N_{\min}}} \operatorname{sgn}\{\alpha\}\operatorname{sgn}\{\sigma\} \\
&\quad\times \prod_{i=1}^{N_{\min}} \|\phi_{\sigma_i}\|_\theta^2\,\delta_{\alpha_i,\sigma_i} \\
&= \frac{K}{N_{\min}!}\sum_{\sigma\in\mathcal{S}_{N_{\min}}} (\operatorname{sgn}\{\sigma\})^2 \prod_{i=1}^{N_{\min}} \|\phi_{\sigma_i}\|_\theta^2 \\
&= K\prod_{n=0}^{N_{\min}-1} \|\phi_n\|_\theta^2
\end{aligned}$$

where we have used the fact that the cardinality of $\mathcal{S}_{N_{\min}}$ is equal to $N_{\min}!$ in obtaining the last equality. This completes the proof of *Theorem 1*. □

Using *Theorem 1*, we immediately obtain the following theorem.

Theorem 2: The SEP for coherent detection of M-ary PSK signals using OC with an N_{A}-element antenna array in the presence of N_{I} uncorrelated equal-power cochannel interferers and thermal noise in Rayleigh fading is given by

$$P_e = \frac{K}{\pi}\int_0^\Theta A(\theta)C(\theta)d\theta. \tag{41}$$

Here, $A(\theta)$ is given by (22), and $C(\theta) \triangleq \prod_{n=0}^{N_{\min}-1}\|\phi_n\|_\theta^2$, with $N_{\min} = \min\{N_{\mathrm{A}},N_{\mathrm{I}}\}$ is the product norm squared of all the elements in a particular orthogonal system generated by $d\mu_\theta(x)$ of (29) using $z(x,\theta) = \psi(x,\theta)$.

Thus, the derivation of the SEP for coherent detection of M-ary PSK using OC, involving the $N_{\min}$-fold integrals in (23), essentially reduces to a simple single integral over θ with finite integration limits. The integrand is a product of two functions $A(\theta)$ and $C(\theta)$; the former function $A(\theta)$ involves trigonometric functions and is given by (22), and the latter function $C(\theta)$ can be evaluated easily as described by the pseudocode provided in Table I, based on the approach illustrated in Appendix A. Finally, the SEP expression (41) can be efficiently and rapidly evaluated using standard mathematical packages, even for a large number of antennas and/or cochannel interferers, where previous studies relied on highly time-consuming simulations.

TABLE I
PSEUDOCODE FOR EVALUATION OF $C(\theta)$

function $C(\theta, N_{\min})$

$C = b_0 = G_0(\theta)$

$\phi_{0,0} = 1$

for $n = 1$ to $N_{\min} - 1$

  for $m = 0$ to $n - 1$

$$\phi_{n,m} = -\sum_{j=m}^{n-1}\frac{\phi_{j,m}}{b_j}\sum_{k=0}^{j}\phi_{j,k}\,G_{n+k}(\theta)$$

  end for

  $\phi_{n,n} = 1$

$$b_n = \sum_{m=0}^{n}\sum_{l=0}^{n}\phi_{n,m}\phi_{n,l}\,G_{m+l}(\theta)$$

  $C = C \cdot b_n$

end for

return C
end

V. NUMERICAL RESULTS

In this section, the performance in terms of SEP of adaptive arrays with OC is investigated using the analytical approach given in *Theorem 2*, with different choices of the SNR defined as E_{D}/N_0, the ratio between the desired received signal power and the total interfering power (SIR) defined as $E_{\mathrm{D}}/(N_{\mathrm{I}}\cdot E_{\mathrm{I}})$, the number of interferers, and the number of antennas.

Let us first assess the accuracy of our analytical tool. It is worthwhile noting that *Theorem 2* provides the exact SEP, under the assumption that the residual interference term at the com-

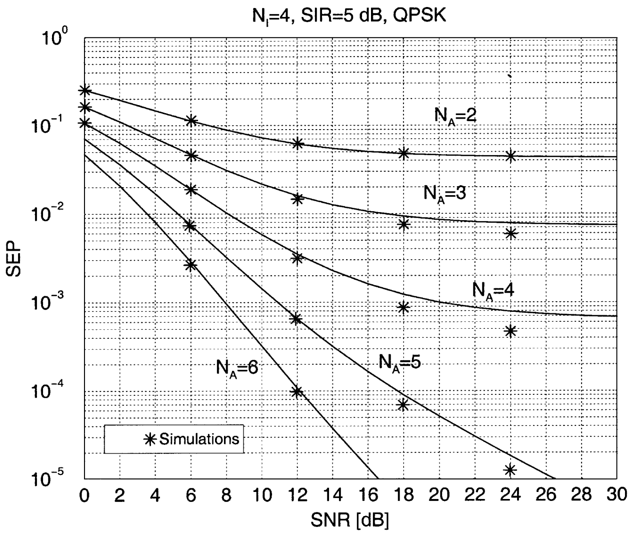

Fig. 1. Comparison between analytical and simulation results, quaternary PSK modulation, $N_{\rm I} = 4$ interferers.

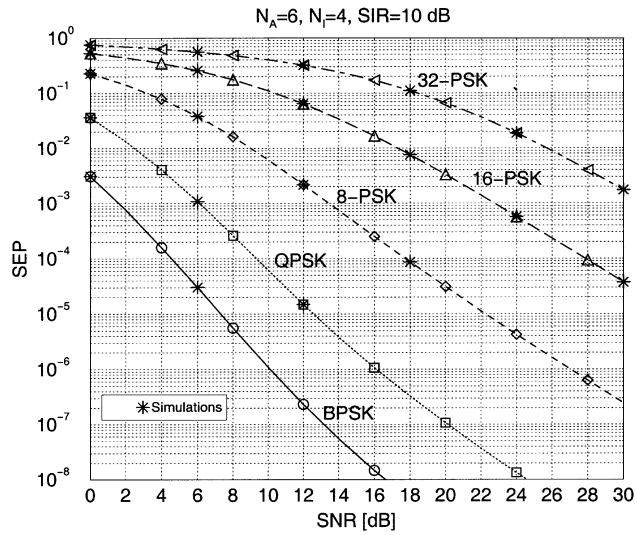

Fig. 2. SEP as a function of SNR for $N_{\rm A} = 6, N_{\rm I} = 4$, SIR $= 10$ dB. Several modulation formats are considered: binary PSK, quaternary PSK, 8-PSK, 16-PSK, and 32-PSK. Semianalytical results are also provided (symbols).

biner output is a Gaussian random variable [27] leading to (20).[4] This is verified in Fig. 1 where the analytical results are compared to time-consuming simulation results obtained for asynchronous cochannel interferers.[5] We remark that here the simulation is at the signal level, thus without any adjoint hypothesis. The simulation, which took several days of computation time, was obtained with at least 1000 error events per point and can be considered quite reliable. The goodness of the exact analytical results based on the Gaussian approximation can be appreciated in the figure and justify the adoption of the Gaussian model for the residual interference after combining.

Fig. 2 shows the SEP as a function of SNR for $N_{\rm A} = 6$ antenna branches, $N_{\rm I} = 4$ interfering signals, and SIR $= 10$ dB. Several modulation formats are considered: binary PSK, quaternary PSK, 8-PSK, 16-PSK, and 32-PSK. Also shown in the figure are semianalytical results obtained by generating the random propagation vectors, computing the SINR by (9), and then averaging the conditional SEP given by [4, eq. (15)]. Since the analytical framework proposed in this paper provides an exact result under the same hypotheses, we find perfect agreement between our analysis and semianalytical results.

Fig. 3 shows the SEP for coherent detection of 8-PSK with five antenna branches as a function of SNR for SIR of 10 dB, with the number of interfering signals ranging from one to eight. It can be seen that when the number of interferers becomes equal to or larger than the number of receiving antennas, the curve exhibits an error floor. This can be easily explained by remembering that adaptive array systems have $N_{\rm A} - 1$ degrees of freedom to cope with interfering signals and thermal noise. When the number of interferers is greater than the array degrees of freedom, the system is not able to null out the interferers and, for large values of SNR, the performance is limited by the interfering power. The opposite is true when the number of antenna branches is greater than the number of interferers; here the additional $L_{\rm Div} = N_{\rm A} - N_{\rm I}$ degrees of freedom are used to mitigate thermal noise and desired signal multipath fading, and this provides an asymptotic behavior for SEP proportional to $1/({\rm SNR})^{L_{\rm Div}}$ (in other words, a diversity degree $L_{\rm Div}$ with respect to fading of the useful signal).

[4]This was always assumed, explicitly or implicitly, in all previous literature on OC [2], [8]–[15].

[5]Fig. 1 shows the SEP as low as 10^{-5} only to illustrate the asymptotic behaviors of the SEP; these extremely low SEPs are not practical, especially for wireless mobile communications. Similar comments apply to the extremely low SEP ranges shown in Figs. 2 and 3.

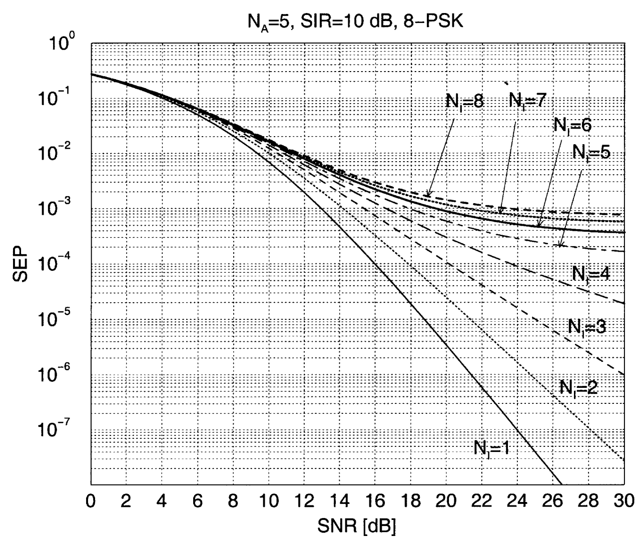

Fig. 3. SEP as a function of SNR for $N_{\rm A} = 5$, 8-PSK, SIR $= 10$ dB; the number of interferers ranges from 1 to 8.

Fig. 4 shows the required SNR as a function of SIR to achieve a target value of SEP of 10^{-3} for the case of 8-PSK in the presence of four interfering signals. The number of antennas ranges from two to six. The figure shows, in the plane (SNR, SIR), the pair of points which provide the desired SEP for a given number of antennas and interferers; the regions above the curves represent the values of SNR and SIR, providing a SEP less than the target SEP. For example, if we fix SNR = 14 dB, an optimum linear combiner with six receiving antennas requires a SIR of about 9 dB to achieve a target SEP of 10^{-3}. This value increases to 14 dB with $N_{\rm A} = 5$ and 21 dB with $N_{\rm A} = 4$; moreover, when the number of antenna branches is smaller than four, no

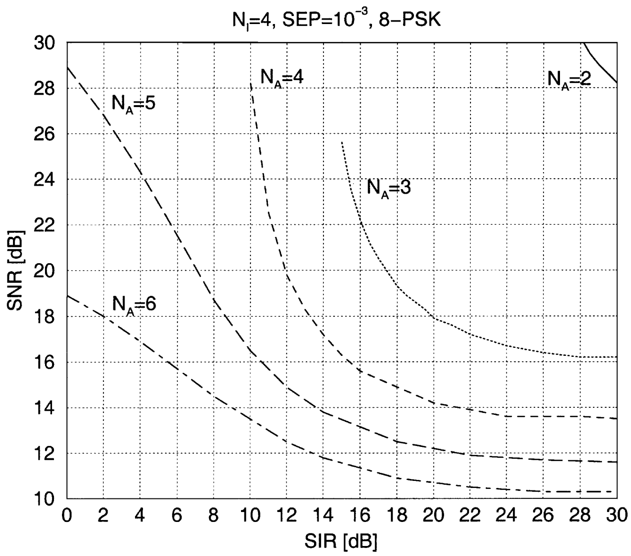

Fig. 4. SNR as a function of SIR, desired SEP $= 10^{-3}$, $N_{\mathrm{I}} = 4$, 8-PSK; N_{A} ranges from 2 to 6.

value of SIR can achieve the desired SEP. A similar behavior can be observed if we fix SIR and determine for the required value of SNR. Finally, when SIR $\to \infty$, the result for OC converges to those for MRC, the performance is limited only by thermal noise; this justifies the floor observed, whatever the value of N_{A}.

VI. Conclusions

In this paper, we have derived the exact SEP for coherent detection of M-ary PSK using optimum combining in the presence of multiple uncorrelated equal-power interferers and thermal noise in a flat Rayleigh fading environment. Starting from an expression requiring the numerical evaluation of nested integrals, and by using the theory of orthogonal polynomials, we obtained a simple and numerically stable solution with only one finite-limit integral, valid for M-ary PSK and an arbitrary number of interferers and/or antenna elements. This made possible the exact SEP evaluation for wireless systems with many users and antennas, where previous studies relied on highly time-consuming simulations. Hence, performance evaluation of wireless systems scenarios with optimum combining, that were either extremely time consuming or impossible by simulation with current computing power, becomes feasible.

Appendix

Derivation of the Orthogonal System

As pointed out earlier in Section IV, the polynomials $1, x, x^2, \ldots, x^{N_{\min}-1}$ in the Hilbert Space $\mathcal{P}_\theta^{N_{\min}}$, with inner product $\langle f, g\rangle(\theta)$ and norm $\|f\|_\theta$ given by (30) and (31), respectively, are linearly independent. Therefore, we can apply the Gram–Schmidt procedure to obtain the orthogonal systems as follows.

First, polynomial 1 gives the function $\phi_0(x,\theta)$ as follows:

$$1 \longrightarrow \phi_0(x,\theta) = 1. \tag{42}$$

The polynomial x produces the second function by

$$x \longrightarrow \phi_1(x,\theta) = x - \frac{\langle x, \phi_0(x,\theta)\rangle}{\|\phi_0\|_\theta^2}\phi_0(x,\theta). \tag{43}$$

In general, the polynomial x^n for $n = 0, \ldots, N_{\min} - 1$ transforms into

$$x^n \longrightarrow \phi_n(x,\theta) = x^n - \sum_{j=0}^{n-1} \frac{\langle x^n, \phi_j(x,\theta)\rangle}{\|\phi_j\|_\theta^2}\phi_j(x,\theta). \tag{44}$$

Now, adopting the following notation for polynomials:

$$\phi_n(x,\theta) = \phi_{n,0}(\theta) + \phi_{n,1}(\theta)x + \cdots + \phi_{n,n}(\theta)x^n \tag{45}$$

the norm square of $\phi_n(x,\theta)$ can be expressed as

$$\|\phi_n\|_\theta^2 = \sum_{l=0}^{n}\sum_{m=0}^{n} \phi_{n,l}(\theta)\phi_{n,m}(\theta)\left\langle x^l, x^m\right\rangle. \tag{46}$$

Using the inner product (30) with $z(x,\theta) = \psi(x,\theta)$, $\|\phi_n\|_\theta^2$ becomes

$$\|\phi_n\|_\theta^2 = \sum_{l=0}^{n}\sum_{m=0}^{n} \phi_{n,l}(\theta)\phi_{n,m}(\theta)G_{l+m}(\theta). \tag{47}$$

where

$$G_k(\theta) \triangleq \int_0^\infty x^{k+N_{\max}-N_{\min}} e^{-x}\psi(x,\theta)dx. \tag{48}$$

A closed-form expression for $G_k(\theta)$ can be derived starting from the integrals [31, eq. 3.353.5]

$$\begin{aligned}\int_0^\infty x^k e^{-x}\frac{x+a}{x+a+b}dx = k! - b\Bigg[&(-1)^{k-1}(a+b)^k \\ &\times e^{a+b}\mathrm{Ei}(-a-b) + \sum_{m=1}^{k}(m-1)!(-a-b)^{k-m}\Bigg]\end{aligned} \tag{49}$$

where $\mathrm{Ei}(\cdot)$ is the exponential integral function [31, Sec. 8.2]. Using (49) in (48) and the relations between the exponential integral function and the incomplete Gamma function $\Gamma(a,z)$ [31, Sec. 8.35], we finally have

$$\begin{aligned}G_{k-N_{\max}+N_{\min}}(\theta) = \zeta(\theta)^k e^{\zeta(\theta)} k! \Bigg[&\zeta(\theta)(1+k) \\ &\times \Gamma\left(-1-k, \zeta(\theta)\right) + \frac{N_0}{E_{\mathrm{I}}}\Gamma\left(-k, \zeta(\theta)\right)\Bigg]\end{aligned} \tag{50}$$

where $\zeta(\theta) = c_{\mathrm{MPSK}}E_{\mathrm{D}}/(\sin^2\theta E_{\mathrm{I}})$.

The coefficients $\phi_{n,m}(\theta)$ can be calculated iteratively using the following formula, which we derive as follows. Substituting (45) into (44) gives

$$\begin{aligned}\phi_n(x,\theta) =& x^n - \sum_{j=0}^{n-1}\frac{1}{\|\phi_j\|_\theta^2}\left\langle x^n, \sum_{k=0}^{j}\phi_{j,k}(\theta)x^k\right\rangle \\ &\times \sum_{m=0}^{j}\phi_{j,m}(\theta)x^m \\ =& x^n - \sum_{j=0}^{n-1}\frac{1}{\|\phi_j\|_\theta^2}\sum_{k=0}^{j}\phi_{j,k}(\theta)\left\langle x^n, x^k\right\rangle \\ &\times \sum_{m=0}^{j}\phi_{j,m}(\theta)x^m.\end{aligned}$$

Once again, using the inner product (30) with $z(x,\theta)=\psi(x,\theta)$, we have

$$\phi_n(x,\theta)=x^n-\sum_{m=0}^{n-1}\left[\sum_{j=m}^{n-1}\frac{1}{\|\phi_j\|_\theta^2}\phi_{j,m}(\theta)\times\sum_{k=0}^{j}\phi_{j,k}(\theta)G_{n+k}(\theta)\right]x^m. \quad (51)$$

Comparing (45) and(51), we obtain the mth coefficient of the nth polynomial as

$$\phi_{n,m}(\theta)=-\sum_{j=m}^{n-1}\frac{1}{\|\phi_j\|_\theta^2}\phi_{j,m}(\theta)\sum_{k=0}^{j}\phi_{j,k}(\theta)G_{n+k}(\theta)\qquad m=0,\ldots,n-1 \quad (52)$$

with $\phi_{n,n}(\theta)=1$.

Exploiting the previous relations (47) and (52) leads to the procedure shown in Table I.

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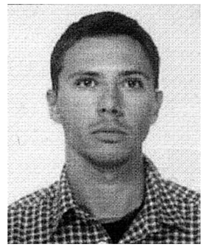

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