In other words, if $a \models p$ and $b \equiv_A c$ $(b, c \in B)$

then $b \equiv_{Aa} c$.

<u>Defn</u> Let $\kappa > |T|$. A set $\mathcal{M} \subseteq \mathcal{U}$ is $\kappa$-saturated if

$\forall A \subseteq M$ $|A| < \kappa$, $\forall p \in S(A)$, $p$ is realised in $\mathcal{M}$

<u>Fact</u> $\forall A$ $\exists \mathcal{M} \supseteq A$ s.t. $\mathcal{M}$ is $|T|^+$-saturated.

<u>Lemma</u> Let $\mathcal{M}$ be $|T|^+$saturated, $p \in S(\mathcal{M})$ definable.

Then (i) $p$ has a unique definition (up to equivalence)

(ii) the unique definition is good.

- $\forall B$, let $p|_B$ be the type resulting from the application of the definition to $B$.

(iii). $\forall B \supseteq \mathcal{M}$, $p|_B$ is a nonsplitting extension of $p$.

<u>Proof</u> (i) Assume $\{d_\varphi p\}$ and $\{d'_\varphi p\}$ are both definitions & not equivalent.

So $\exists \varphi$ s.t. $d_\varphi p \not\equiv d'_\varphi p$.

ie $\exists b$ (not in $\mathcal{M}$) s.t. say $\models d_\varphi p(b)$ & $\not\models d'_\varphi p(b)$

~~So there exists~~

So $tp(b/\mathcal{M})$ contradicts over $d_\varphi' p(y)$.

$\Rightarrow \exists c \in \mathcal{M}$ & $\chi(y,z)$ s.t. $\models \chi(b,c)$ and

$\chi(y,c)$ contradicts $d_\varphi' p(y)$.

Let $A$ = set of parameters used in $d_\varphi p$, then

$A \subseteq \mathcal{M}$, $|A| \leq |T|$.

By $|T|^+$-saturation $\exists b' \in \mathcal{M}$ st. $b' \equiv_{Ac} b$.

Then $\models d_\varphi p(b')$ & $\not\models d_\varphi' p(b')$ (because $\chi(b',c)$).

So $d_\varphi p$, $d_\varphi' p$ do not define the same $\varphi$-type in $\mathcal{M}$.

(ii) Let $B$ be any set.

We want to prove $p|_B$ is a complete consistent type.

~~consistent~~ Let $A$ be, as above, the set of parameters.

consistent: if not, there are $\varphi_i(x, b_i) \in p|_B$ $i<n$ st.

$\bigwedge \varphi_i(x, b_i)$ is inconsistent.

By saturation, find $\overline{b'} \equiv_A \overline{b}$, $\overline{b'} \in \mathcal{M}$.

Then $\varphi_i(x, b_i') \in p$ $\forall i$ and $\bigwedge \varphi_i(x, b_i')$ is inconsistent

$\#$

complete: Assume not. Then $\exists b \in B$ and $\varphi(x,y)$ s.t.

$\varphi(x,b) \notin p|_B$ and $\forall \psi$ contradicting $\varphi$, $\psi(x,b) \notin p|_B$.

find $b' \underset{A}{\equiv} b$ in $\mathcal{M}$ ....etc..

(iii) not enough time, so exercise!