

# 18.S096: Topics in Mathematics with Applications in Finance

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# 18.S096: Key Facts

- The purpose of the class is to expose undergraduate and graduate students to the mathematical concepts and techniques used in the financial industry.
- Mathematics lectures will be mixed with lectures illustrating the corresponding application in the financial industry.
- MIT mathematicians will teach the mathematics part while industry professionals will give the lectures on applications in finance.
- We plan to organize an optional field trip to visit Morgan Stanley offices in New York on October 15.
- Prerequisites: 18.01, 18.02, 18.03 (or approval of instructor), 18.05 or 18.440, 18.06
- Prior knowledge of economics or finance is not required but may be helpful for some lectures.
- The class will have problem sets for each math lecture (due in 2 weeks after assignment) and a final paper on a math finance topic of student's choice.
- There will be no exams. The final grade will be 75% based on the homework and 25% on final paper.

# How are Financial Products Traded?

Introduction of basic knowledge of financial markets and quantitative analysis

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MIT Course 18.S096

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**The views represented herein are the author's own views and do not necessarily represent the views of his past and current employers.**

The Story of  
Pi,  
Alpha,  
Beta,  
Delta,  
Gamma,  
Theta,

and

Vega

Markets, Exchanges, OTC, Regions

Money, Currencies

Stocks, Equity Indices, IPOs,  
Primary/Secondary

Loans, Bonds, Credit Products

Commodity

Real Estate, Mortgages, ABS

Derivatives, Structured Products

Banks: deposits, commercial lending

Investment Banks:

Fixed Income

Equity

IBD, Corporate Finance

Asset Management

Wealth Management

Zero-sum game?

Investors—have money, need return

Borrowers—need money, have risks

Trade between lender and borrower is  
still the main driving force.

## Market Participants:

Dealers, Market Makers, Brokers

Individuals, Retail Investors

Mutual Funds, Insurance Companies, Pension Funds, Asset Managers, SWFs

Hedge Funds (various strategies), PE

Governments, Policy Makers

Corporate Hedgers, Liability Management

## Trader Types:

1. Hedger

2. Market Maker, bid and offer

3. Proprietary Trader

Fund Portfolio Manager

beta and alpha, linear regression of  
two time series: return of assets a  
and b:

$$R(a) = \alpha + \beta * R(b)$$

## Hedging Trade Examples:

Bond issuer hedge interest rate exposures,  
hedge of payables.

Example: 10yr Samurai bonds issued in JPY by  
Australia corporates. Fixed rate difference of 4.5% -  
1% = 3.5%. Need receive JPY and pay AUD to hedge.  
FX, swaps and cross-currency basis.

Corporates/producers hedge revenue.

Currency hedge of future receivables

Importer and exporter hedge FX

Deal contingent hedges

## Market Making Examples:

Simple product bid/offer, price transparency, liquidity

Inventory of risks, manage a dealer book, balancing Greeks: delta, gamma, theta, vega, tail risks

Value at Risk (VaR), Capital, Balance Sheet, Risk Weight Assets

Proprietary Trading Examples:

Directional, long, short,

Arbitrage

Value, Relative Value

Systematic, momentum, statistical  
arbitrage

Fundamental Analysis. Global Macro

Special situations, distressed

# Financial Mathematics:

## 1. Pricing Models:

relative value, arbitrage-free.

## 2. Risk Management:

human risk aversion, greed/fear.

## 3. Trading Strategies:

“Holy Grail” strategy, perpetual motion machine, Robo-Trader.

Risk aversion:

Choice A: 80% chance to lose \$500, 20% chance to win \$500.

Choice B: 100% chance to lose \$280.

Which choice?

Now

Choice A: 80% chance to win \$500, 20% chance to lose \$500.

Choice B: 100% chance to win \$280.

Which Choice?

Control factor. Marginal utility.

Do people always learn from their experiences?

Long market cycle and short memory

Historical extrapolation.

Deterministic vs statistic.

Efficient Market Theory and behavioral finance.

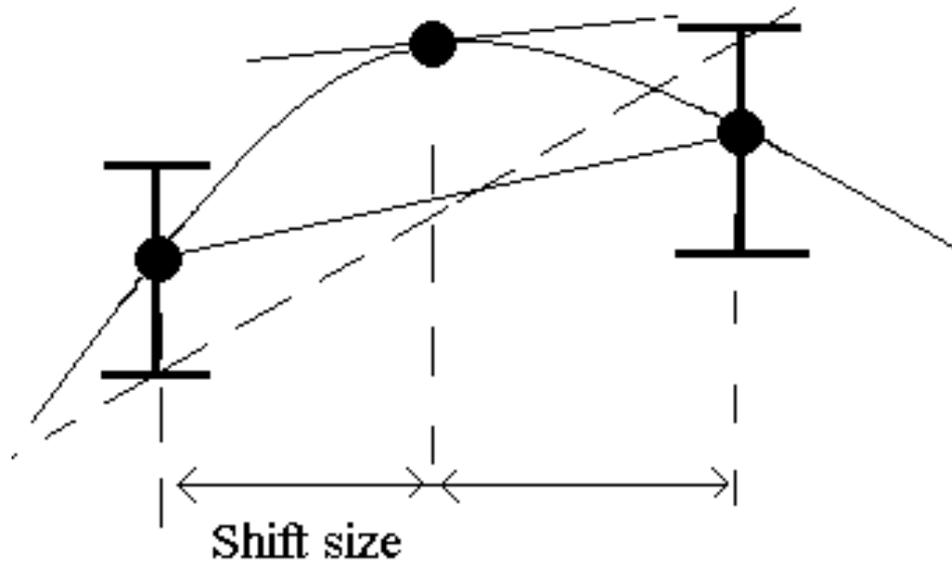
Over-simplification.

Home work assignment (optional):

Make your own financial glossary (the terminology you don't understand) by searching the web or reading the materials on the course website.

# Project 1 – Noisy Delta

- Monte Carlo pricing
- Delta computation with centered difference
- Question: What is the optimal shift size?



Courtesy of Dusan Milijancevic

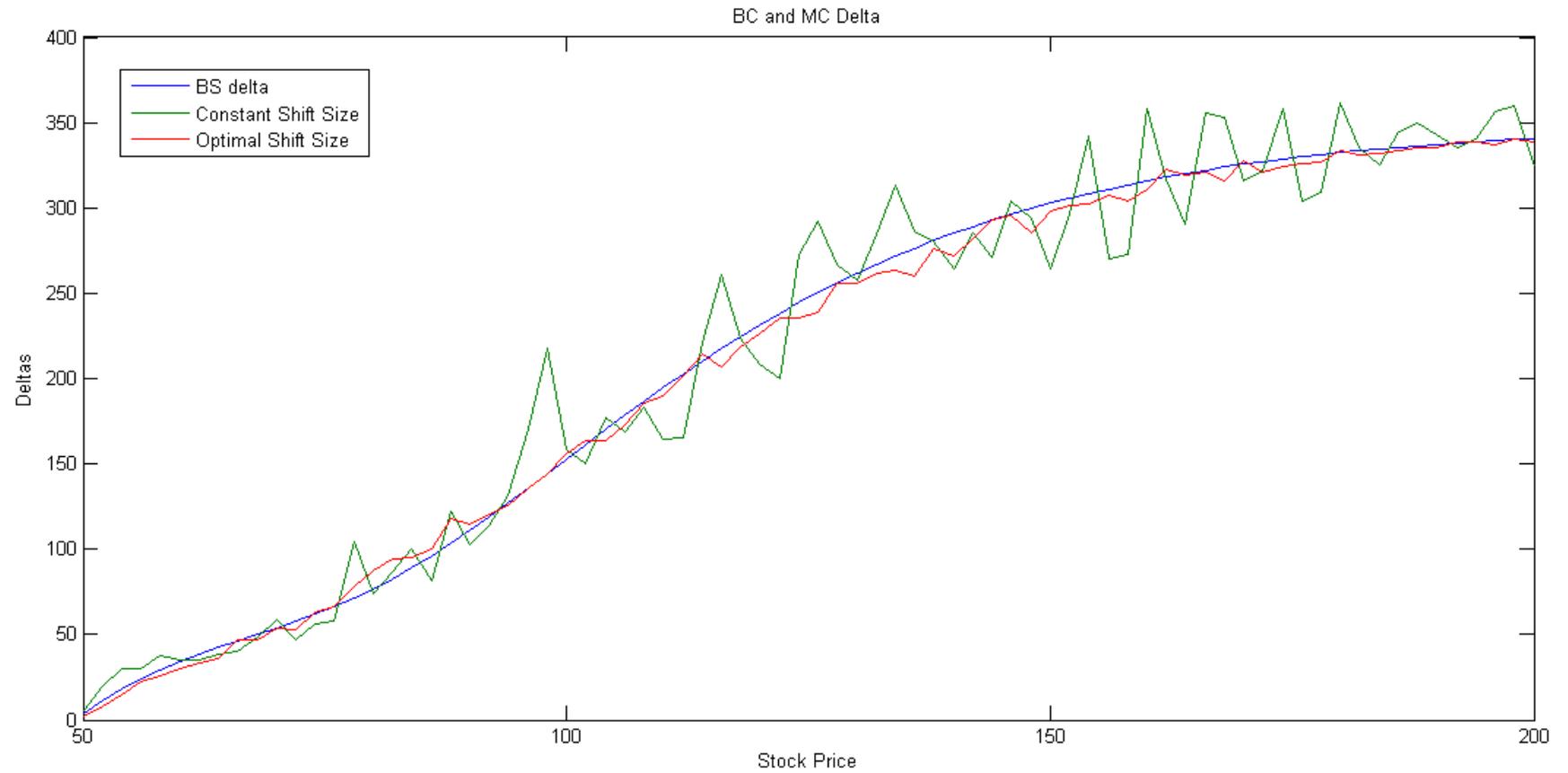
# Project 1 – Noisy Delta

- For the solution we minimize the standard error

$$E\left(\left|f'(x) - \frac{\hat{f}(x + \varepsilon) - \hat{f}(x - \varepsilon)}{2\varepsilon}\right|^2\right)$$

$$\varepsilon_{opt} = \frac{1}{\sqrt[3]{2c}} \sqrt[3]{\frac{sd(x)}{d + |\Gamma(x)|}}$$

# Project 1 – Noisy Delta



Courtesy of Dusan Milijancevic

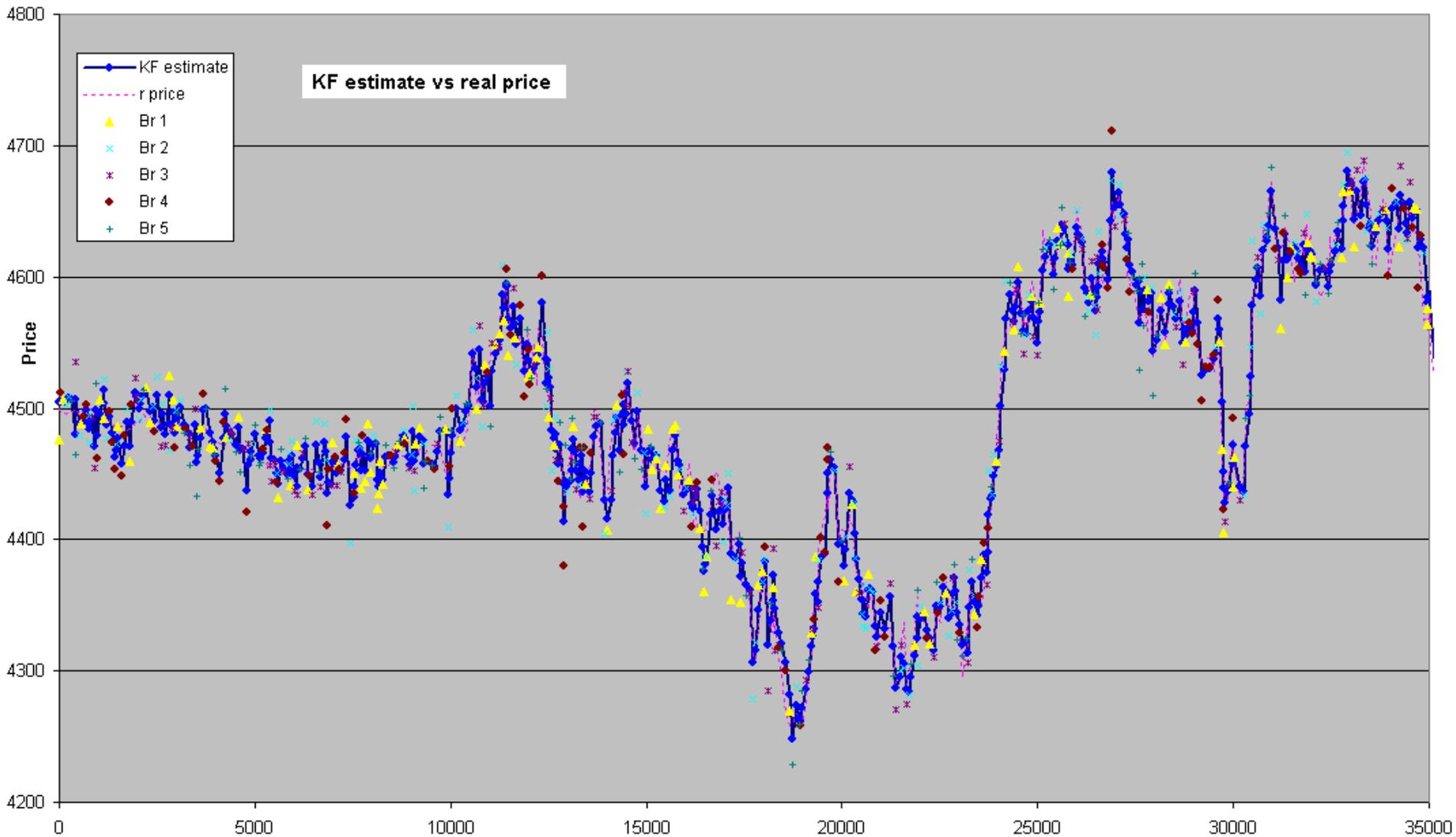
# Project 2 – Kalman Filter

- Five brokers quote swap points for USDRUB forward contract at different times
- The aim of the project is to study if Kalman filter can improve accuracy of quotes

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19/03/13 8:31:04	4718	19/03/13 7:56:25	4725	19/03/13 8:03:57	4750	19/03/13 9:13:49	4750	19/03/13 8:05:28	4675
19/03/13 8:37:35	4721	19/03/13 8:01:18	4730	19/03/13 8:40:12	4750	20/03/13 8:22:31	4730	19/03/13 8:05:35	4680
19/03/13 8:53:35	4723	19/03/13 8:01:25	4725	19/03/13 12:05:01	4750	20/03/13 11:31:38	4720	19/03/13 8:16:55	4685
19/03/13 9:05:22	4731	19/03/13 8:03:54	4740	19/03/13 12:26:51	4760	21/03/13 6:36:38	4820	19/03/13 9:09:47	4690
19/03/13 9:20:54	4728	19/03/13 9:46:26	4740	19/03/13 12:27:05	4750	21/03/13 7:50:17	4840	19/03/13 14:09:29	4685
19/03/13 10:49:28	4726	19/03/13 9:46:30	4740	19/03/13 12:27:23	770	21/03/13 7:52:47	4860	19/03/13 15:53:37	4690
19/03/13 13:01:00	4728	19/03/13 11:54:01	4740	19/03/13 12:27:25	4760	21/03/13 8:28:24	4850	19/03/13 15:59:17	4695
19/03/13 13:01:57	4731	19/03/13 12:09:15	4745	19/03/13 12:34:18	4760	21/03/13 8:33:55	4860	19/03/13 16:03:44	4700
19/03/13 16:02:40	4738	19/03/13 12:33:53	4745	19/03/13 12:34:26	4750	21/03/13 9:37:37	4880	19/03/13 16:11:14	4705
19/03/13 16:15:52	4758	19/03/13 12:46:25	4750	19/03/13 12:50:08	4760	22/03/13 6:44:31	4775	19/03/13 16:21:07	4710
20/03/13 7:08:27	4738	19/03/13 13:18:51	4750	19/03/13 13:16:34	4755	22/03/13 8:00:46	4790	20/03/13 5:54:19	4715
20/03/13 7:32:59	4743	19/03/13 13:24:35	4740	19/03/13 13:16:52	4755	22/03/13 8:04:17	4785	20/03/13 5:54:23	4725
20/03/13 7:35:10	4733	19/03/13 13:24:41	4745	19/03/13 13:22:08	4750	22/03/13 8:16:49	4790	20/03/13 7:19:45	4720
20/03/13 7:40:31	4728	19/03/13 13:25:41	4745	19/03/13 13:22:10	4755	26/03/13 7:01:38	4760	20/03/13 7:20:01	4715
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Courtesy of Dusan Milijanecvic

# Project 2 – Kalman Filter



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