PROFESSOR: All right, let's start. So first of all, I hope you've been enjoying the class so far. And thank you for filling out the survey. So we got some very useful and interesting feedbacks. One of the feedbacks is my impression, I haven't gotten a chance to talk to my co-lecturers or colleagues yet, but I read some comments.

You was saying that some of the problem sets are quite hard. The math part may be a bit more difficult than the lecture. So I'm thinking. So is really the application lecture. And from now, after three more lectures by [INAUDIBLE], it will be essentially the remainder is all applications. The original point of having this class is really to show you how math is applied, to show you those cases in different markets, different strategies, and in the real industry.

So I'm trying to think, how do I give today's lecture with the right balance? This is, after all, a math class. Should I give you more math, or should I-- you've had enough math. I mean, it sounded like from the survey you probably had enough math. So I would probably want to focus a bit more on the application side. And from the survey also it seems like most of you enjoyed or wanted to listen to more on the application side.

So anyway, as you've already learned from Peter's lecture, the so-called Modern Portfolio Theory. And it's actually not that modern anymore, but we still call it Modern Portfolio Theory. So you probably wonder, in the real world, how actually we use it. Do we follow those steps? Do we do those calculations?

And so today, I'd like to share with you my experience on that, both in the past, a different area, and today probably more focused on the buy side. Oh, come on in. Yeah. Actually, these are my colleagues from Harvard Management. So--

[CHUCKLES]

--they will be able to ask me really tough questions. So anyway, so how I'm going to start this class. You wondered why I handed out to each of you a page. So does everyone have a blank page by now? Yeah, actually. Yeah. Could also pass to--? Yeah. So I want every one of you to use that blank page to construct a portfolio, OK?

So you're saying, well, I haven't done this before. That's fine. Do it totally from your intuition, from your knowledge base as of now. So what I want you to do is to write down, to break down the 100% of what do you want to have in your portfolio. OK, you said, give me choices. No, I'm not going to give you choices. You think about whatever you like to put down. Wide open, OK?

And don't even ask me the goal or the criteria. Base it on what you want to do. And so totally free thinking, but I want you to do it in five minutes. So don't overthink it. And hand it back to me, OK? So that's really the first part. I want you to show intuitively how you can construct a portfolio, OK?

So what does a portfolio mean? That I have to explain to you. Let's say for undergraduates here, so your parents give you some allowance. So you manage to save a $1,000 on the side. You decided to put into your investments, buying stocks or whatever, or gambling, buy lottery tickets, whatever you can do. Just break down your percentage.

That could be $1,000, or you could be a portfolio manager and have hundreds of billions of dollars, or whatever. Or then and say if they raise some money, start a hedge fund, they may have $10,000 just to start with. How do you want to use those money on day one? Just think about it. And then so while you're filling out those pages, please hand it back to me. It's your choice to put your name down or not.

And then I will start to assemble those ideas and put them on the blackboard. And sometimes I may come back to ask you a question-- you know, why did you put this? That's OK. Don't feel embarrassed. We're not going to put you on the spot.

But the idea is I want to use those examples to show you how we actually connect theory with practice. I remember when I was a college student I learned a lot of different stuff. But I remember one lecture so well, one teacher told me one thing. I still remember vividly well, so I want to pass it on to you.

So how do we learn something useful, right? You always start with observation. So that's kind of the physics side. You collect the data. You ask a lot of questions. You try to find the patterns. Then what you do, you build models. You have a theory. You try to explain what is working, what's repeatable, what's not repeatable.

So that's where the math comes in. You solve the equations. Sometimes in economics, lot of times, unlike physics, the repeatable patterns are not so obvious. So what you do after this, so you come back to observations again. You confirm your theory, verify your predictions, and find you error. Then this feeds back to this rule.

And a lot of times, the verification process is really about understanding special cases. That's why today I really want to illustrate the portfolio theory using a lot of special cases. So can you start to hand back your portfolio construction by now? OK, so just hand back whatever you have. If you have one thing on the paper, that's fine. Or many things on the paper, or you think as a portfolio manager, or you think as a trader, or you think simply as a student, as yourself.

All right, so I'm getting these back. I will start to write on the blackboard. And you can finish what you started.

By the way, that's the only slide I'm going to use today. I'm not concerned-- you realize if I show you a lot of slides, you probably can't keep up with me. So I'm going to write down everything, just take my time. And so hopefully you get a chance to think about questions as well.

OK, I think-- is anyone finished? Any more? OK. All right, OK. OK, great. You guys are awesome. OK, so let me just have a quick look to see if I missed any, OK?

Wow, very interesting. So I have to say, some people have high conviction. 100% of you, one of those. I think I'm not going to read your names, so don't worry, OK?

OK I'm just going to read the answers that people put down, OK? So small cap equities, bonds, real estate, commodities. Those were there. Qualitative strategies, selection strategies, deep value models. Food/drug sector models, energy, consumer, S&P index, ETF fund, government bonds, top hedge funds. So natural resources, timber land, farmland, checking account, stocks, cash, corporate bonds, rare coins, lotteries, collectibles. That's very unique. And Apple's stock, Google stock, gold, long term saving annuities.

So Yahoo, Morgan Stanley stocks. I like that.

[LAUGHTER]

OK. Family trust. OK, I think that pretty much covered it. OK, so I would say that list is more or less here. So after you've done this, when you were doing this, what kind of questions came to your mind? Anyone wants to-- yeah, please.

AUDIENCE: [INAUDIBLE] how do I know what's the right balance to draw in my portfolio? Whether it would be cash, bills, or stuff like that?

PROFESSOR: How do you do it, really? What's the criteria? And so before we answer the question how you do, how do you group assets or exposures or strategies or even people, traders, together-- before we ask all those questions, we have to ask ourselves another question. What is the goal? What is the objective, right?

So we understand what portfolio management is. So here in this class, we're not talking about how to come up with a specific winning strategy in trading or investments. But we are talking about how to put them together. So this is what portfolio management is about.

So before we answer how, let's see why. Why do we do it? Why do we want to have a portfolio, right? That's a very, very good point. So let's understand the goals of portfolio management. So before we understand goals of portfolio management, let's understand your situations, everyone's situation.

So let's look at this chart. So I'm going to plot your spending as a function of your age. So when you are age 0 to age 100, so everyone's spending pattern is different. So I'm not going to tell you this is the spending pattern. So obviously when kids are young, they probably don't have a lot of hobbies or tuition, but they have some basic needs. So they spend.

And then the spending really goes up. Now your parents have to pay your tuition, or you have to borrow-- loans, scholarships. And then you have college. Now you have-- you're married. You have kids. You need to buy a house, buy a car, pay back student loans. You have a lot more spending.

Then you go on vacation. You buy investments. You just have more spending coming up. So but it goes to a certain point. You will taper down, right? So you're not going to keep going forever. So that's your spending curve. And with the other curve, you think about it. It's what's your income, what's your earnings curve. You don't owe anything where you are just born.

I use earning. So this is spending. So let's call this 50. Your earning probably typically peaks around age 50, but it really depends. Then you probably go down, back up. Right, so that's your earning. And do they always match well? They don't.

So how do you make up the difference? You hope to have a fund, an investment on the side, which can generate those cash flows to balance your earning versus your spending. OK, so that's only one simple way to put it. So you've got to ask about your situation. What's your cash flow look like?

So my objective is, I'm going to retire at age of 50. Then after the age of 50, I will live free. I'll travel around the world. Now I'll calculate how much money I need. So that's one situation. The other situation is, I want to graduate and pay back all the student loans in one year. So that's another. And typically people have to plan these out.

And if I'm managing a university endowment, so I have to think about what the university's operating budget is like, how much money they need every year drawing from this fund. And then by maintaining, protecting the total fund for basically a perpetual purpose, right? Ongoing and keep growing it. You ask for more contributions, but at the same time generating more return.

If you have a pension fund, you have to think about what timeframe lot of the people, the workers, will retire and will actually draw from the pension. And so every situation is very different. Let me even expand it. So you think, oh, this is all about investment. No, no, this is not just about investment.

So I was a trader for a long time at Morgan Stanley, and later on a trade manager. So when I had many traders working for me, the question I was facing is how much money I need to allocate to each trader to let them trade. How much risk do they take, right? So they said, oh, I have this winning strategy. I can make lots of money. Why don't you give me more limits?

No, you're not going to have all the limits. You're not going to have all the capital we can give to you. Right, so I'm going to explain. You have to diversify. At the same time, you have to compare the strategies with parameters-- liquidity, volatility, and many other parameters. And even if you are not managing people, let's say-- I was going to do this, so Dan, [INAUDIBLE], Martin and Andrew. So they start a hedge fund together.

So each of them had a great strategy. Dan has five, Andrew has four, so they altogether have 30 strategies. So they raise an amount of money, or they just pool together their savings. But how do you decide which strategy to put more money on day one? So those questions are very practical. So that's all. So you understand your goals, that's then you're really clear on how much risk you can take.

So let's come back to that. So what is risk? As Peter explained in his lecture, risk is actually not very well defined. So in the Modern Portfolio Theory, we typically talk about variance or standard deviation of return. So today I'm going to start with that concept, but then try to expand it beyond that. So stay with that concept for now. Risk, we use standard deviation for now. So what are we trying to do?

So this, you are familiar with this chart, right? So return versus standard deviation. Standard deviation is not going to go negative. So we stop at zero. But the return can go below zero. And I'm going to review one formula before I go into it. I think it's useful to review what previously you learned.

So you let's say you have-- I will also clarify the notation as well so you don't get confused. So let's say-- so Peter mentioned Harry Markowitz' Modern Portfolio Theory which won him the Nobel Prize in 1990, right? Along with Sharp and a few others. So it's a very elegant piece of work.

But today, I will try to give you some special cases to help you understand that. So let's review one of the formulas here, which is really the definition. So let's say you have a portfolio. Let's call the expected return of the portfolio is R of P equal to the sum, a weighted sum, of all the expected returns of each asset. You'll basically linearly allocate them.

Then the variance-- oh, let's just look at the variance, sigma p squared. So these are vectors. This is a matrix. The sigma in the middle is a covariance matrix. , OK that's all you need to know about mass at this point.

So I want us to go through an exercise on that piece of paper I just collected back to put your choice of the investment on this chart. OK, so let's start with one. So what is cash? Cash has no standard deviation. You hold cash-- so it's going to be on this axis. It's a positive return. So that's here.

So let's call this cash. Where is-- and let's just about it. Where's lottery? Say you buy Powerball, right? So what is lottery for? Let's assume you put everything in lottery. So you're going to lose. So your expected value is very close to lose 100%. And your standard deviation is probably very close to 0. So you will be here.

So some of you say, oh, no, no. It's not exactly zero. So OK, fine. So maybe it's somewhere here, OK? So not 100%, but you still have a pretty small deviation from losing all the money. What is coin flipping? So let's say you decide to put all your money to gamble on a fair coin flip, fair coin. So expected return is zero. What is the standard deviation of that?

AUDIENCE: 100%?

PROFESSOR: Good. So 100%. So we got the three extreme cases covered. OK, so where is US government bond? So let's just call it five-year note or ten-year bond. So the return is better than cash with some volatility. Let's call it here.

What is investing in a start up venture capital fund like? Pretty up there, right? So you'll probably get a very high return, by you can lose all your money. So probably somewhere here, you see. Buying stocks, let's call it somewhere here. Our last application lecture, you heard about investing in commodities, right? Trading gold, oil. So that has higher volatility, so sometimes high returns. So let's call this commodity.

And the ETF is typically lower than single stock volatility, because it's just like index funds. So here. Are there any other choices you'd like to put on this map? OK. So let me just look at what you came up with.

Real estate, OK. Real estate, I would say probably somewhere around here. Private equity probably somewhere here. Or investing in hedge funds somewhere. So I think that's enough examples to cover. So now let me turn the table around and ask you a question. Given this map, how would you like to pick your investments?

So you learned about the portfolio theory. As a so-called rational investor, you try to maximize your return. At the same time, minimize your standard deviation, right? i hesitate to use the term "risk," OK? Because as I said, we need to better define it. But let's just say you try to minimize this but maximize this, the vertical axis.

OK, so let's just say you try to find the highest possible return for that portfolio with the lowest possible standard deviation. So would you pick this one? Would you pick this one? OK, so eliminate those two. But for this, that's actually all possible, right? So then that's where we learn about the Efficient Frontier.

So what is the Efficient Frontier? It's really the possible combinations of those investments you can push out to the boundary that you can no longer find another combination given the same standard deviation. You can no longer find a higher return. So you reach the boundary. And the same is true that for the same return, you can no longer minimize your standard deviation by finding on other combination. OK, so that's called Efficient Frontier.

How do you find the Efficient Frontier? That's what essentially those work were done and it got them the Nobel Prize, obviously. It's more than that, but you get the flavor from the previous lectures. So what I'm going to do today is really reduce all of these to the special case of two assets. Now we can really derive a lot of intuition from that.

So we have sigma R. We're going to ignore what's below this now, right? We don't want B there. And we want to stay on the upright. So let's consider one special case. So again for that, let's write out for the two assets. So what is R of p? It's w1 R1 plus y minus w1 R2, right? Very simple math. And what is sigma P?

So the standard deviation of the portfolio-- or the variance of that, which is a square-- we know that's for the two asset class special case. So let me give you a further restriction-- which, let's consider if all R1 equal to R2. Again, here meaning expected return. I'm simplifying some of the notations. And sigma y equal to 0, and sigma 2 not equal to 0, so what is rho? What is the correlation?

0, right? Because you have no volatility on it. OK, so what is-- what's that?

AUDIENCE: It's really undefined.

PROFESSOR: It's really undefined, yes. Yeah.

AUDIENCE: [INAUDIBLE] no covariance.

PROFESSOR: There's no-- yeah, that's right. OK, so let's look at this. So you have sigma 2 here. Sigma 1 is 0. And you have R1 equal to R2. What is all R of P? It's R, right? Because the weighting doesn't matter. So you know it's going to fall along this line. So here is when weight y equal to 0. So you weigh everything on the second asset.

Here you weigh the first asset 100%. So you have a possible combination along this line, along this flat line. Very simple, right? I like to start with a really a simple case. So what if sigma 1 also is not 0, but sigma 1 equal to sigma 2. And further, I impose-- impose-- the correlation to be 0, OK?

What is this line look like? So I have sigma 2 to equal to sigma 1. And R1 is still equal to R2, so Rp is still equal to R1 or R2, right? What does this line look like? So volatility is the same. Return is those are the same of each of the asset class. You have two strategies or two instruments.

They are serially correlated. How would you combine them? So you take the derivative of this variance with regarding to the weight, right? And then you minimize that. So what you find is that this point is R1 equal to 0, or-- I'm sorry, w1 or [? wy ?] equal to 1. You're at this point, right? Agreed? So you choose either, you will be ending up in portfolio exposure in terms of return, and variance will be right here.

But what if you choose-- so when you try to find the minimum variance, you actually end up-- I'm not going to do the math. You can do it afterwards. You check by yourself, OK? You will find at this point, that's when they are equally weighted, half and half. So you get square root of that. So you actually have a significant reduction of the variance of the portfolio by choosing half and half, serially correlate the portfolio.

So what's that called? What's that benefit? Diversification, right? Where you have less than perfectly correlated, positively correlated assets. You can actually achieve the same return by having a lower standard deviation. I'll say, OK, that's fairly straightforward. So let's look at a few more special cases. I want really to have you establish this intuition.

So let's think about what if in the same example, what if rho equal to 1 perfectly correlated? Then you can't, right? So you end up at just this one point. You agree? OK.

What if it's totally negatively correlated? Perfectly negatively correlated. What's this line look like? Right? So you if you weight everything to one side, you're going to still get this point. But if you weight half and half, you're going to achieve basically zero variance. I think we showed that last time, you learned that last time.

OK, so let's look beyond those cases. So what now? Let's look at-- so R1 does not equal to R2 anymore. Sigma 1 equal to 0. There's no volatility of the first asset. So that's cash, OK? So that's a riskless asset in the first one.

So let's even call that R1 is less than R2. So that's the-- right? You have the cash asset, and then you have a non-cash asset. Rho equal to 0, zero correlation. So let's look at what this line looks like. So R1, R2, sigma 2 here.

When you weight assets to 100%, you're going to get this point, right? When you weight asset one 100%, you're going to get this point, right? So what's in the middle of your return as a function of variance? Can someone guess?

AUDIENCE: A parabola? Should it be a parabola?

PROFESSOR: Try again.

AUDIENCE: A parabola.

PROFESSOR: Yeah, I know, I know. Thank you. Are there any other answers? OK, this is actually I-- let me just derive very quickly for you. Sigma 1 equal to 0, rho equal to 0. What's sigma p? Right? And sigma p is essentially proportional to sigma 2 with the weighting.

OK, and what's R? R is a linear combination of R1 and R2. So it's still-- so it's linear. OK, so because in these cases, you actually-- you essentially-- your return is a linear function. And the slope, what is the slope of this?

Oh, let's wait on the slope. So we can come back to this. This actually relates back to the so-called capital market line or capital allocation line, OK? Because last time we talked about the Efficient Frontier. That's when we have no riskless assets in the portfolio, right? When you add on cash, then you actually can select. You can combine the cash into the portfolio by having a higher boundary, higher Efficient

Frontier, and essentially a higher return with the same exposure.

So let's look at a couple more cases, then I will tell you-- so I think let's look at-- so R1 is less than R2. And volatilities are not 0. Also, sigma 1 is less than sigma 2, but it has a negative correlation of 1. So you'll have asset one, asset two. And as we know, where you pick half and half, this goes to 0.

So this is a quadratic function. You can verify and prove it later. And what if when rho is equal to 0-- and actually, I want to-- so sigma 1 should be here, OK? So when rho is equal to zero, this no longer goes to-- the variance can no longer be minimized to 0. So this is your Efficient Frontier, this part.

I think that's enough examples of two assets for the Efficient Frontier. So you get the idea. So then what if we have three assets? So let me just touch upon that very quickly. If you have one more asset here, essentially you can solve the same equations. And when the [INAUDIBLE] special case-- you can verify afterwards-- all the volatilities are equal, and zero correlation among the assets.

You're going to be able to minimize sigma p equal to 1 over the square root of three of sigma 1. OK. So it seems pretty neat, right? The math is not hard and straightforward. But it gives you the idea how-- to answer your question-- how to select them when you start with two. So why are two assets so important? What's the implication in practice?

It's actually a very popular combination. Lot of the asset managers, they simply benchmark to bonds versus equity. And then one famous combination is really 60/40. They called it a 60/40 combination. 60% in equity, 40% in bonds. And even nowadays, any fund manager, you have that. People will still ask you to compare your performance with that combination.

So the two asset examples seem to be quite easy and simple, but actually it's a very important one to compare. And that will lead me to get into the risk parity discussion. But before I get to risk parity discussion, I want to review the concept of beta and the Sharpe ratio.

So your portfolio return, this is your benchmark return of m, expected return. Rf is the risk free return, so essentially a cash return. And alpha is what you can generate additionally. So let's even not to worry about these small other terms-- or not necessarily small, but for the simplicity. And I'll just reveal that. So that's your beta.

Now what is your Sharpe ratio? OK. And you can-- so sometimes Sharpe ratio is also called risk weighted return, or risk adjusted return. And how many of you have heard of Kelly's formula? So Kelly's formula basically gives you that when you have-- let's say in the gambling example, you know your winning probability is p.

So this basically tells you how much to size up, how much you want to bet on. So it's a very simple formula. So you have a winning probability of 50/50, how much you bet on? Nothing. So if you have p equal to 100%, you bet 100% of your position. If you have a winning probability of negative 100%, so what does it mean? That means you have a 100% probability of losing it. What do you do?

You bet the other way around, right? You bet the other side, so that when p is equal to negative-- I'm sorry, actually what I should say is when p equal to 0, your losing probability becomes 100%, right? So you bet 100% the other way, OK?

So that I leave to you to think about. That's when you have discrete outcome case. But when you construct a portfolio, this leads to the next question. It's in addition to the Efficient Frontier discussion, is that really all about asset allocation? Is that how we calculate our weights of each asset or strategy to choose from?

The answer is no, right? So let's look at a 60/40 portfolio example. So again, two asset stock. Stock is, let's say, 60% percent, 40% bonds. So on this-- so typically your stock volatility is higher than the bonds, and the return, expected return, is also higher. So your 60/40 combinations likely fall on the higher return and the higher standard deviation part of the Efficient Frontier.

So the question was-- so that's typically what people do before 2000. A real asset manager, the easiest way or the passive way is just to allocate 60/40. But after

2000, what happened was when after the equity market peaked and the bond has a huge rally as first Greenspan cut interest rates before the Y2K in the year 2000. You think it's kind of funny, but at that time everybody worried about the year 2000. All the computers are going to stop working because old software were not prepared for crossing this millennium event.

So they had to cut interest rates for this event. But actually nothing happened, so everything was OK. But that left the market with plenty of cash, and those after the tech bubble bursted. So that was a good portfolio, but then obviously in 2008 when the equity market crashed, the bond market, the government bond hybrid market, had a huge rally.

And so that made people question that. Is this 60/40 allocation of asset simplify by the market value the optimal way of doing it, even though you are falling on the Efficient Frontier? But how do you compare different points? Is that simple choice of your objectives, your situation, or there's actually other ways to optimize it.

So that's where the risk parity concept was really-- the concept has been around, but the term was really coined in 2005, so quite research, by a guy named Edward Qian. He basically said, OK, instead of allocating 60/40 based on market value, why shouldn't we consider allocating risk? Instead of targeting a return, targeting asset amount-- let's think about a case where we can have equal weighting of risk between the two assets. So risk parity really means equal risk weighting rather than equal market exposure.

And then the further step he took was he said, OK. So this actually, OK, is equal risk. So you have lower return and a lower risk, a lower standard deviation. But sometimes you will really want a higher return, right? How do you satisfy both? Higher return and lower risk. Is there a free lunch? So he was thinking, right?

There is, actually. It's not quite free, but it's the closest thing. You've probably heard this phrase many times. The closest thing in investment to a free lunch is diversification. OK, and so he's using a leverage here as well. let me talk about it a bit more, about diversification, give you a couple more examples, OK? That phrase about the free lunch and diversification was actually from-- was that from Markowitz? Or people gave him that term.

OK, but anyway. So let me give you another simple example, OK? So let's consider two assets, A and B. In year one, A goes up to-- it basically doubles. And in year two, it goes down 50%. So where does it end up?

So it started with 100%. It goes up to 200%. Then it goes down 50% on the new base, so it returns nothing, right? It comes back. So asset B in year one loses 50%, then doubles, up 100% in year two. So asset B basically goes down to 50% and it goes back up to 100%. So that's when you look at them independently.

But what if you had a 50/50 weight of the two assets? So if someone who is quick on math can tell me, what does it change? So A goes up like that, B goes down like that. Now you have a 50/50 A and B. So let's look at magic.

So in year one, A, you have only 50%. So it goes up 100%. So that's up 50% on the total basis. B, you'll also weight 50%, but it goes down 50%. So you have lost 25%. So at the end of year one, you're actually-- so this is a combined 50/50 portfolio, year one and year two.

So you started with 100. You're up to 1.25 at this point, OK? So at the end of year one, you rebalance, right? So you have to come back to 50/50. So what do you do? So this becomes 75, right? So you no longer have the 50/50 weight equal. So you have to sell A to come back to 50 and buy some money to buy B.

So you have a new 50/50 percent weight asset. Again, you can figure out the math. But what happens in the following year when you have this move, this comes back 50%, this goes up 100%. You return another 25% positively without volatility. So you have a straight line. You can keep-- so this two year is a-- so that's so-called diversification benefit.

And in the 60/40 bond market, that's really the idea people think about how to combine them. And so let me talk a little bit about risk parity and how you actually achieve them. I'll try to leave plenty of time for questions.

So that's the return, and so let's forget about these. So let's leave cash here, OK? So the previous example I gave you, when you have two assets, one is cash, R1, the other is not. The other has a volatility of sigma 2. You have this point, right? So and I said, what's in between? It's a straight line. That's your asset allocation, different combination.

Did it occur to you, why can't we go beyond this point? So this point is when we weight w2 equal to 1, w1 equal to 0. That's when you weight everything into the asset two. What if you go beyond that? What does that mean?

OK. So let's say, can we have w1 equal to minus 1, w2 equal to plus 2? So they still add up to 100%. But what's negative 1 mean? Borrow, right? So when you're short cash 100%, you borrow money. You borrow 100% of cash, then put into to buy equity or whatever, risky assets, here. So you have plus 2 minus 1.

What does the return looks like when you do this? So Rp equal to w1 R1 plus w2 R2. So minus R1 plus 2 R2. That's your return. It's this point here. What's your variance look like, or standard deviation look like? As we did before, right?

So sigma p simply equal to w2 sigma 2. So in this case, it's 2 sigma 2. So you're two times more risky, two times as risky as the asset two. So this introduces the concept of leverage. Whenever you go short, you introduce leverage.

You actually on your balance sheet, you have two times of asset two. You're also short one of the other instrument, right? OK so that's your liability. So that is year one. So what this risk parity says is, OK, so we can target on the equal risk weighting, which will give you somewhere around-- let's called it 25. 25% bonds, 75%-- 25% equity, 75% of fixed income. Or in other words, 25% of stocks, 75% of bonds. So you have lower return.

But if you leverage it up, you actually have higher return, higher expected return, given the same amount of standard deviation you achieved by leveraging up. Obviously, you leverage up, right? That's the other implication of that. We haven't talked about the liquidity risk, but that's a different topic.

So what's your Sharpe ratio look like for risk parity portfolio? So you essentially maximized the Sharpe ratio, or risk adjusted return, by achieving the risk parity portfolio. So 60/30 is here. You actually maximize that, and this is-- does leverage matter? When you leverage up, does Sharpe ratio change, or not?

AUDIENCE: It splits in half. So you've got twice the [? variance ?] [INAUDIBLE].

PROFESSOR: So let's look at that straight line, this example, OK? So we said Sharpe ratio equal to-- right? So Rp, what is sigma P? It's 2 sigma 2, right? When you leverage up, so this equals to R2 minus R1. Divide by sigma 2.

So that's the same as you at this point. So that's essentially the slope of the whole line. It doesn't change. OK, so now you can see the connection between the slope of this curve and the Sharpe ratio and how that links back to beta. So let me ask you another question. When the portfolio has higher standard derivation of sigma P, will beta to a specific asset increase or decrease?

So what's the relationship intuitively between beta? So let's take a look at the 60/40 example. Your portfolio, you have stocks, you have bonds in it. So I'm asking you, what is really the beta of this 60/40 portfolio to the equity market? When equity market, it becomes-- when the portfolio becomes more volatile. Is your beta increasing or decreasing?

So you can derive that. I'm going to tell you the result, but I'm not going to do the math here. So beta equals to-- [INAUDIBLE] in this special case, is sigma P over sigma 2. OK. All right, so so much for all these. I mean, it sounds like everything is nicely solved. And so coming back to the real world, and let me bring you back, OK?

So are we all set for portfolio management? We can program, make a robot to do this. Why do we need all these guys working on portfolio management? Or why do we need anybody to manage a hedge fund? You can just give money, right? So why do you need somebody, anybody, to put it together?

So before I answer this question, let me show you a video.

[VIDEO PLAYBACK]

[HORN BLARING]

[END VIDEO PLAYBACK]

OK. Anyone heard about the London Millennium Bridge? So it was a bridge built around that time and thought it had the latest technology. And it would really perfectly absorb-- you heard about soldiers just marching across a bridge, and they'll crush the bridge. When everybody's walking in sync, your force gets synchronized. Then the bridge was not designed to take that synchronized force, so the bridge collapsed in the past.

So when they designed this, they took all that into account. But what they hadn't taken into account was the support of that is actually-- so they allow the horizontal move to take that tension away. But the problem is when everybody's sees more people walking in sync, then the whole bridge starts to swell, right? Then the only way to keep a balance for you standing on the bridge is to walk in sync with other people.

So that's a survival instinct. And so I got this-- I mean, that's actually my friend at Fidelity, Ren Cheng. Dr. Ren Cheng brought this up to me. He said, oh, you're doing-- how do you think about the portfolio risk, right? This is what happened in the financial market in 2008. When you think you got everything figured out, you have the optimal strategy.

When everybody starts to implement the same optimal strategy for your own as individual, the whole system is actually not optimized. It's actually in danger. Let me show you another one.

[VIDEO PLAYBACK]

[CLACKING]

OK. These are metronomes, right? So can start anywhere you like. Are they in sync? Not yet. What is he doing? You only have to listen to it. You don't have to see it. So what's going on here? This is not-- metronomes don't have brains, right? They don't really follow the herd. Why are they synchronizing?

OK, if you're expecting they are getting out of sync, it's not going to happen. OK, so I'm going to stop right here. OK.

[END VIDEO PLAYBACK]

You can try as many-- how do I get out of this? OK, so you can try it. You can look at-- there's actually a book written on this as well, so. But the phenomena here is nothing new. But what when he did this, what's that mean? When he actually raised that thing on the plate and put it on the Coke cans? What happened? Why is that is so significant?

AUDIENCE: Because now they're connected.

PROFESSOR: They're connected. Right. So they are interconnected. Before, they were individuals. Now they're connected. And why did I show you the London Bridge and this at the same time? What's this to do with portfolio management? What's this to do with portfolio management?

AUDIENCE: [INAUDIBLE] people who are trading, if they have the same strategy, [INAUDIBLE] affect each other, they become connected in that way--

PROFESSOR: Right.

AUDIENCE: If as an individual, you are doing a different strategy, if everybody has been doing something different, you can maximize [? in the space. ?]

PROFESSOR: Very well said. So if you're looking for this stationary, best way of optimizing your portfolio, chances are everybody else is going to figure out the same thing. And eventually, you end up in the situation and you actually get killed. OK, so that's the thing. What you learned today, what you walk away was this. OK, today is not what I want you to know that all the problems are solved. Right?

So you say, oh, the problem's solved. The Nobel Prize was given. So let's just program them. No, you actually-- it's a dynamic situation. You have to. So that makes the problem interesting, right? As a younger generation, you're coming to the field. The excitement is there are still a lot of interesting problems out there unsolved. You can beat the others already in the field. And so that's one takeaway.

And what are the takeaways you think by listening to all these?

AUDIENCE: Diversification is a free lunch.

[CHUCKLES]

PROFESSOR: Diversification is a free lunch, yes. Not so free, right, in the end. It's free to a certain extent. But it's something-- you know, it's better than not diversified, right? It depends on how you do it. But there is a way you can optimize. And so it's-- I want to leave with you, I actually want to finish a few minutes earlier so that you can ask me questions. You can ask. It's probably better to have this open discussion.

And so I want you to walk away to really keep in mind is in the field of finance, and particularly in the quantitative finance, it's not mechanical. It's not like solving physics problems. It's not like you can get everything figured so it becomes predictable, right? So the level of predictability is actually very much linked to a lot of other things.

Physics, you solve Newton's equations. You have a controlled environment and you know what you're getting in the outcome. But here, when you participate in the market, you are changing the market. You are adding on other factors into it. So think more from a broader scope kind of view rather than just solve the mathematics.

That's why I come back to the original. If you walk away from this lecture, you'll remember what I said at the very beginning. Solving problems is about observe, collecting data, building models, then verify and observe again. OK, so I'll end right here, so questions.

AUDIENCE: Yeah, just [INAUDIBLE] question. Does this have anything to do with-- it kind of sounds like game theory, but I'm not exactly too sure. Because you have a huge population and no stable equilibrium. Does it have anything to do with game theory, by any chance?

PROFESSOR: It has a lot to do with game theory, but not only to game theory. So game theory, you have a pretty well-defined set of rules. Two people play chess against each other. That's where a computer actually can become smarter, right?

So in this market association, you have so many people participating without clearly defined rules. There are some rules, but not always clearly defined. And so it's much more complex than game theory. But it's part of it, yeah. Dan, yeah?

AUDIENCE: Can you talk a little bit about loss in the risk parity portfolios that did so poorly in May and June when rates started to rise and what about their portfolio allowed them do that?

PROFESSOR: Good question, right. So as you can see here, what the risk parity approach does is essentially to weight more on the lower volatility asset. In this case, the question is, how do you know which asset has low volatility? So you look at historical data, which you conclude bonds have the lower volatility. So you overweight bonds. That's the essence of them, right?

So then when bonds to start to sell off after Bernanke, Fed chairman Bernanke, said he's going to taper quantitative easing. So bonds from a very low high yield, a very low yield level. The yield went much higher, the interest rate went higher. Bonds got sold off. So this portfolio did poorly.

So now the question is, does that prove the risk parity approach wrong, or does it prove right? Does the financial crisis of 2008 prove the risk parity approach a superior approach, or does the June/May experience prove this as the less-favored approach? What does it tell us?

Think about it. So it really is inconclusive. So you observe, you extrapolate from your historical data. But what you are really doing is you're trying to forecast volatility, forecast return, forecast correlation, all based on historical data. It's like a lot of people use that example. It's like driving by looking at the rearview mirror. That's the only thing you look at, right? You don't know what's going on, happening in front of you. You have another question?

AUDIENCE: Given all this new information, do you find that people are still playing similar [INAUDIBLE] strategy with portfolio management?

PROFESSOR: Very much true. Why? Right, so you said, people should be smarter than that. It's very difficult to discover new asset classes. It's also very difficult to invent new strategies in which you have a better winning probability. The other risk, the other very interesting phenomena, is most of the traders and the portfolio managers, the investors, they are career investors-- meaning just like if I'm a baseball coach, I'm hired to coach a baseball team. My performance is really measured against the other teams when I win or lose, right?

A portfolio manager or investor is also measured against their peers. So the safest way for them to do is to benchmark to an index, to the herd. So there's very little incentive for them to get out of the crowd, because if they are wrong, they get killed first. They lose their jobs. So the tendency is to stay with the crowd. It's for survival instinct.

It's, again, the other example. It's actually the optimal strategy for individual portfolio manager is really to do the same thing as other people are doing because you stay with the force.

AUDIENCE: So you said given that we have all these groups, in the end, it's not just that we could leave it to the computers. We need managers. So what different are the managers doing, other than [INAUDIBLE]?

PROFESSOR: Can you try to answer that question yourself? What's the difference between a human and a computer? That's really-- what can human add value to what a computer can do?

AUDIENCE: Consider the factors, the market factors and news and what's going on.

PROFESSOR: So taking more information, processing information, make a judgment on a more holistic approach. So it's an interesting question. I have to say that computers are beating humans in many different ways. Can a computer ever get to the point actually beating a human in investment? I can't confidently tell you that it's not going to happen. It may happen. So I don't know.

Any other questions? Yeah?

AUDIENCE: Just to add to that. I think there is some more to management than just investing. I think managers also have key roles in their HR, key roles in just like managing people and ensuring that they're maximizing their talents, not just like, oh, how much money did you make? But I mean, are you moving forward in your career while you're there? So I think management has a role to play in that as well, not just investment.

PROFESSOR: Yeah, I think that's a good point. Yeah. All right, so-- oh, sure. Jesse?

AUDIENCE: What is your portfolio breakdown?

PROFESSOR: My personal portfolio? Well, I am actually very conservative at this point, because if you look in my curve of those spending and earning curve, I'm basically trying to protect principles rather than try to maximize return at this point. So I would be sliding down more towards this part rather than try to go to this corner, yeah.

So I haven't really talked much about risk. What is risk, right? So I talk about volatility or standard deviation. But as we all know that, as Peter mentioned last time as well, there are many other ways to look at risk-- value at risk or half distribution or truncated distribution, or simply maximum loss you can afford to take, right?

But looking at a standard or volatility is an elegant way. You can see. I can really show you in very simple math about how the concept actually plays out. But in the end, actually volatility is really not the best measure, in my view, of risk. Why? Let me give you another simple example before we leave.

So let's say this is over time. This is your cumulative return or you dollar amount. So you start from here. If you go flat, then-- does anyone like to have this kind of a performance? Right? Of course, right? This is very nice. You keep going up. You never go down.

But what's the volatility of that? The volatility is probably not low, right? And then on the other hand, you could have-- what I'm trying to say, when you look at expected return matching expected return and the volatility, you can still really not selecting the best combination. Because what you really should care about is not just your volatility.

And again, bear in mind all the discussion about the Modern Portfolio Theory is based on one key assumption here. It's about Gaussian distribution, OK? Normal distribution. The two parameters, mean and standard deviation, categorize the distribution. But in reality, you have many other sets of distributions. And so it's a concept still up for a lot of discussion and debate. But I want to leave that with you as well. Yeah?

AUDIENCE: Just going back to the same question about what these guys were asking about management and how do they add value, I think the people who added value-- there were some people who added a tremendous amount of value in the financial crisis. And they were doing the same mathematics. But a difference was in their expected return of various assets was different from the entire-- the broad market. So if you can just know what expected return is that, probably that is the only answer to the whole portfolio management debate.

PROFESSOR: Yes. If you can forecast expected return, then that't-- yeah, now you know the game. You solved it. You solved the big part of the puzzle. Yeah?

AUDIENCE: And management does is how good it can do [INAUDIBLE] expected return, full stop. Nothing more.

PROFESSOR: I disagree on that. That's the only thing. Because given two managers, they have the same expected return, but you can still further differentiate them, right? So that's-- yeah. And that's what all this discussion is about. But yes, expected return will drive lot of these decisions. If you know one manager's good expected return, three years later, he's going to make 150%. You don't really care what's in between, right? You're just going to ride it through. But the problem is you don't know for sure. You will never be sure.

AUDIENCE: I'd like to comment on that.

PROFESSOR: Sure.

AUDIENCE: What [INAUDIBLE] looked at in simplified settings, estimating returns and volatilities. And the problem, the conclusion for the problem, was basically cannot estimate returns very well, even with more data, over a historical period. But you can estimate volatility much better with more data. So there's really an issue of perhaps luck in getting the return estimates right with different managers, which are hard to prove that there was really an expertise behind that. Although with volatility, you can have improved estimates.

And I think possibly with a risk parity portfolio, those portfolios are focusing not on return expectations, but saying if we're going to consider different choices based on just how much risk they have and equalize that risk, then the expected return should be comparable across those, perhaps.

PROFESSOR: Yeah. So that highlights the difficulty of forecasting return, forecasting volatility, forecasting correlation. So risk parity appears to be another elegant way of proposing the optimal , strategy but it has the same problems. Yeah?

AUDIENCE: Actually, I also wanted to highlight. You mentioned the Kelly criterion, which we haven't covered the theory for that previously. But I encourage people to look into that. It deals with issues of multi-period investments as opposed to single period investments. And most all this classical theory we've been discussing, or that I discuss, covers just a single period analysis, which is an oversimplification of an investment.

And when you are investing over multiple periods, the Kelly criterion tells you how to optimally basically bet with your bank roll. And actually there's an excellent book, at least I like it, called *Fortune's Formula* that talks about-- [? we already ?] said the origins of options theory in finance. But it does get into the Kelly criterion.

And there was a rather major discussion between Shannon, a mathematician at MIT, who advocated applying the Kelly criterion, and Paul Samuelson, one of the major economists.

PROFESSOR: Also from MIT.

AUDIENCE: Also from MIT. And there was a great dispute about how you should do portfolio optimization.

PROFESSOR: That's a great book. And a lot of characters in that book actually are from MIT-- and Ed Thorp, for example. And it's really about people trying to find the Holy Grail magic formula-- not really to that extent, but finding something other people haven't figured out. But it's very interesting history. Big names like Shannon, very successful in other fields.

In his later part of his career and life really devoted most of his time to studying this problem. You know Shannon, right? Claude Shannon? He's the father of information theory and has a lot to do with the later information age invention of computers and very successful, yeah. So anyway, so we'll end the class right here. No homework for today, OK? So you just need to-- yeah, OK. All right, thank you.