DENIS GOROKHOV: So I work at Morgan Stanley. I ran corporate treasury strategies at Morgan Stanley. So corporate treasury is the business unit that is responsible for insuring and risk management of Morgan Stanley debt. around their strategists own the New York inflation desk. That's the business which is a part of the global interest rate business, which is responsible for trading derivatives linked to inflation.

And today, I'm going to talk about the HJM model. So HJM model-- the abbreviation stands for Heath-Jarrow-Morton, these three individuals who discovered this framework in the [INAUDIBLE]. And this is a very general framework for pricing, derivatives to interest rates, and to credit.

So on Wall Street, big banks make a substantial amount of money by trading all kinds of things-- exotic products, exotic derivatives. And big banks like Morgan Stanley, like Goldman, JP Morgan-- trades thousands and thousands of different types of exotic derivatives. So a typical program which the business faces is that new types of derivatives arrive all the time.

So you need to be able to respond quickly to the demand from the clients. And you need to be able not just to tell the prices derivative. You need to be able also to risk manage this derivative. Because let's say if you sold an option, you've got some premium. If something goes more in your favor, you need to pay in the end. So you need to be able to hedge.

And you can think about the HJM model, like this kind of framework, as something which is similar to theoretical physics in a way, right? So you get beautiful models-- it's like a solvable model. For example, let's say the hydrogen atom in quantum mechanics.

So it's relatively straightforward to solve it, right? So we have an equation, which can be exactly solved. And we can find energy levels and understand this fairly quickly.

But if you start going into more complex problems-- for example, you add one more electron and you have a helium atom-- it's already much more complicated. And then if you have complicated atoms or even molecules, it's unclear what to do. So people came up with approximate that kind of method, which allow nevertheless solve like very accurately numerically.

And HJM is a similar framework. So you can-- it allows to price all kinds of [INAUDIBLE] derivatives. And so it's very general. It's very flexible to incorporate new payoffs, all kinds of correlation between products and so on, so forth.

And this HJM model-- [INAUDIBLE] natural [INAUDIBLE] more general framework like Monte Carlo simulation. And before actually going into details of pricing exotic interest rates and [INAUDIBLE] derivatives, let me just first explain how this framework appears in the most common type of derivatives [INAUDIBLE] product.

So like a very, very simple example, right? So let's say if we cover derivatives, they ask that [INAUDIBLE], and they sell all kinds of products. Of course, ideally, let's say there's a client who wants to buy something from you. Of course, the easiest approach would be to find the client and do an opposite transaction with him, so that you're market neutral, at least in theory. So if you don't want to count counterparties and so on.

However, it's rather difficult in general, so the portfolios are very complicated. And there's always some residual risk. So there's [INAUDIBLE] of dynamic hedging. So for this example, very simple example, a dealer just sold a call option on a stock. And if you do this, then the principle-- the amount of money which you can lose is unlimited.

So you need to be able to hedge dynamically by trading underlying, for example, in this case. So just this [INAUDIBLE] of the stock markets, you see how random it has been for the last 20 years or so. So first of all, this year, some kind of from beginnings of the 1990s to around 2000, we see [INAUDIBLE] a very sharp increase. And then we have dot-com bubble, and then we have the bank increase of 2008.

And if you trade derivatives whose payoff depends, for example, on [INAUDIBLE] kind of index, you should be very careful. All right? Because market can drop, and you need to be hedged. So you need to be able to come with some kind of good models which can recalibrate to the markets and which can truly risk manage your position.

So the so the general idea of [INAUDIBLE] derivatives is that one starts from some stochastic process. So in this example here, it's probably like the simplest possible-- nevertheless a very instructive-- model, which is essentially like these [INAUDIBLE], which is where we have the stock, which follows the normal dynamics.

A couple questions. Do you have a pointer somewhere, or not? It's just easier-- OK, OK.

PROFESSOR: Let's see. There's also a pen here, where you can use this.

DENIS GOROKHOV: Oh, I see.

PROFESSOR: Have you used this before? You press the color here that you want to use, say, and then you can draw. You press on the screen.

DENIS GOROKHOV: Oh, I see. Excellent. That's even better.

OK, so it seems like the market is very random. We need to be able to come up with some kind of dynamics. And [INAUDIBLE] the normal dynamics is what it is in the first approximation for virtual dynamics.

So in this example, we have selected the differential equation for the stock prize.

And it consists-- it's the sum of two charms, which is [INAUDIBLE]. It's some kind of deterministic part of the stock price dynamics. And here, also, we have diffusion.

So here, dB is the Brownian motion driving the stock, and S is the price of the stock here. Mu is the drift. And sigma is the volatility of the stock. Particularly, it shows the randomness. And it's the randomness impact on the stock price.

And using this model, one can do it as a Black-Scholes formula. And the Black-Scholes formula shows how to price derivatives, whose payoff depends on the price of the stock. So here, if you look at this differential equation, then you can answer the question.

Let's say if you started from some initial value for the stock at time 2. And then we started the clock, which are now to be less at time, capital T. And given that time T, then stock price is ST. So what's the probability distribution for the stock at time T?

So this kind of equation can be very easily solved. And one can up to [INAUDIBLE], their probability distribution function at any crucial moment of time. So I mean, I just think I read a few equations, because it's very important to understand this. So I'm you probably have seen something like this already, but let me just show you like [INAUDIBLE] use beyond this formula.

So if you have a random process-- let's say A is some process, special process, which is normal. So it's full of some drift. Plus some volatility turn. Right? So the difference between this equation is that I don't multiply by A here and A here. Especially, it's much simpler to solve.

So the solution for this equation is very straightforward. So at any moment of time T, if you start at moment 0, the solution of the equation would be something like this. Drift-- right? I'm simply integrating. Plus-- And I assume that B of t is [INAUDIBLE] motion, so at time 0, it's 0.

And then it's very easy to see now that is equal to the Brownian motion. But this is nothing else. It's some random number, which is normally distributed times square root of time. So epsilon is proportional to it.

OK, so basically, this means that this is normally distributed. And it's probability distribution for this quantity is equal to-- we know it's exactly right, because this is like a standard Gaussian distribution. And if you simply substitute A into here, then you will obtain the probability distribution for the actual quantity. And I'll just write it for the completeness.

So basically, we obtain probability distribution for the standard variable. So this is straightforward. So the only difference between the case I'm doing here is that the dynamics is going to be log normal. Right?

And the implication is very simple. If it's normal, then the price of the stock can become negative. This is just a financial [INAUDIBLE]. So the [INAUDIBLE] normal dynamics basically is a good first approximation. And in this case, what helps as a result is just known as [INAUDIBLE]. So I just first of all write it, and then I will explain how you can obtain it.

And if you look at this equation-- let me write it once again-- which basically is a drift plus [INAUDIBLE]. That, of course, seems it intuitively is clear that the dynamics of logarithm of S is dynamic [INAUDIBLE] of S is normal. So essentially, we obtain something like this.

So if you now substitute this into this, you locked in a very simple formula. OK, so here, I used the result, which is known as [INAUDIBLE], which I'm going to explain right now. Like how it was obtained-- basically, it tells us that when we differentiate the function of a stochastic variable.

Then besides the trivial term, which is basically the first derivative times dS. There's an additional term, which is proportional to the second derivative. And it's no stochastic, so I'll explain like this. But if you do it-- if you look at this equation, then you see essentially this formula. It's very, very similar to this formula. The only difference now is that alpha is just mu minus one half of sigma squared.

So that's a [INAUDIBLE]. If you usually use this solution, and since you simply substitute A by log S, you will come to this equation. So this is very important. So it's a very important effect, like-- yes?

AUDIENCE: The fact that it can't be negative, does that exclude certain possibilities? When there's a normal Gaussian, can you [INAUDIBLE] a positive?

DENIS GOROKHOV: Yes, but stock-- from a financial point of view, stock cannot be a liability. Right? You buy a stock. This means basically, you pay some money. And you have basically some sort of, say, option on the profit of the company.

So they can charge you by default. So it can go negative for the stock. Also, in principles, it might be derivatives, which can be both positive or negative payoff, but not the stock. So it's fundamental financial restriction.

So we already [INAUDIBLE]. So if you talk about the stock dynamics and a [INAUDIBLE], it's very important that this [INAUDIBLE] for the stock can be found exactly. And I'll just respond very briefly-- [INAUDIBLE] again, for the Black-Scholes formula, its very important just for understanding. And I believe there are a couple of things which at this point, when I was studying this, it was not very clear to me, so I want to go to some more detail.

So basically, here, this derivation is almost like every textbook. So the idea is that a very fundamental result in stochastic calculus. That if you have a stochastic function-- function of stochastic variable S and time, then its differential can be written as the following form.

So this is all very clear, right? This is standard calculus? It's straightforward. But there is an additional term that looks a bit suspicious. And I will explain what it actually means is on the next slide.

So a very important thing is that when you [INAUDIBLE], then you will [INAUDIBLE] deterministic term is proportional to the second derivative. And you see, there is no-- the fact is that you have here dt, basically this looks like it's an additional contribution to the drift. We wish this is drift, and this is a drift. And there is no [INAUDIBLE].

It's very important. This is like a crucial fact beyond the [INAUDIBLE] and the Monte Carlo method in finance And then [INAUDIBLE], you can [INAUDIBLE], for example, in [INAUDIBLE]. So if we issue an option, then we hedge it-- by having a [INAUDIBLE] position in the underlying.

So the idea is like this. Let's say I sold a call option of the stock. So when the stock market goes, the stock makes some money. And then, in the same time, I short the stock, so I lose money on my hedge. And wherever the market goes, I don't make or lose money. So that's the idea, basically, beyond hedging.

And basically, what happens if I [INAUDIBLE] changes in my portfolio, then since it is no [INAUDIBLE]. I assume I am perfectly hedged. Then I simply obtain [INAUDIBLE]. So out here is a risk-free interest rate. So if you simply look at this equation and substitute the [INAUDIBLE] result here, then you obtain like a very simple equation, which is basically Black-Scholes differential equation for the stock-- for the price of the option.

So this equation is very fundamental. And its very elegant. So you can see although originally, right, if you started from something with some arbitrary risk, with some authority drift mu. Right? Which is basically-- it could be anything.

Which is it? This drift mu drops out of the equation. And it depends on the interest rate. And this is a very interesting fact. So and this very interesting fact has to do with hedging.

Again, you have position in an option, and you have the opposite position in underlying. And that's how the drift disappears. If you look at the movement of both the positions, then you see that there's a drift, it will disappear. So it's a very important and striking fact.

And the second thing, which is truly a miracle, is that risk is omitted completely. So this equation has absolutely no stochasticity. So you can just solve it. If you specify as option payoff, and if you know your volatility, which is a measure of your basically all from how the stock fluctuates. And if you know the [INAUDIBLE] interest rate, you can just price the options.

And this is a true miracle that occurs. And when I was studying this, I couldn't really understand this-- maybe because I was coming from theoretical physics, and all this result [INAUDIBLE] is buried somewhere in stochastic calculus. And I would just try to understand what [INAUDIBLE] means. And let them just explain here, basically how one can understand [INAUDIBLE] is the result is a [INAUDIBLE] in a very simple term. [INAUDIBLE] terms.

So let me just write-- let me remind you. So [INAUDIBLE] basically tells the following-- once again, so if C is the function of stochastic variables, of stochastic variable S, then its differential is not just equal to some standard result from calculus. But we also get some kind of very exotic term, which is basically [INAUDIBLE] nontrivial.

And let me just try to explain to you how [INAUDIBLE] appears. So just to understand this, [INAUDIBLE] everybody after this lecture, look at this derivation, because it still explains what this [INAUDIBLE] means. So the idea is very simple. So let's start for electrons for the first principles. And let's say we have an interval of time, which [INAUDIBLE].

And let's say we divide it into intervals, and each interval lens is dt prime. Right? And I assume that the ratio or dt over dt prime is sufficiently large. So first, we know that our stock, as we know, follows the normal dynamics.

So this means that if I go from from time i to time i plus 1, here you need to exchange i and i minus 1. So then, you can always [INAUDIBLE] right? So S-- if time i plus 1 minus S at time i is equal to the drift term-- right? Which is a discreet version of the stochastic differential equation. Plus the randomness.

So here again, [INAUDIBLE] sigma is volatility. It's the measure of how the stock fluctuates, which is the stock price, which is square root of dt, because Brownian motion fluctuation is proportional to the time. And also here, we have epsilon, and epsilon is a standard, normal variable.

Then-- OK, so we have this. This is pretty straightforward. Basically, I just throw stochastic differential equation on the latest. And I go from point i to point i plus 1.

Now, let's see what it means for the price absorption. So again, so see the price absorption. At time T, when the stock price is equal to Si plus 1. So the change in the option price is equal to-- like the first term, just something very standard, standard calculus.

Plus the first derivatives and the difference in the stock price, plus I take the second total term, which is the second derivative, and I have here S times i plus 1, minus S times i squared. So this is approximate, because I'm taking only the main terms. Or the other terms, given that both times dT, energy prime are very small, they can be neglected.

So you can check it carefully at home if you want to. but But I guarantee that there is no miracle here. Everything we need is here.

Now [INAUDIBLE]. So we have this equation. And let's look at this term. So this term, basically, is a cornerstone of this Ito's Lemma. So let's take this equation for the difference and [INAUDIBLE] them to here.

And you see here, again, you can look-- what is important, it gains a time scales. So dt prime is very small. Therefore, the term, which is random, dominates here. Right? Because [INAUDIBLE] square root of dt. And square root for small times is much bigger than the linear function. Therefore, we simply neglect this term compared this term. And [INAUDIBLE] accuracy in dt prime, we can approximate this just by this term.

Now, what to do-- so again, they're all the same equation, that in [INAUDIBLE] difference for the option price of two neighboring points. And what I'm doing right now, I have all this equation, and I will simply sum them-- basically from 0 to N. So let's say I have all these equations from 0 to N minus 1, and I sum them. And again, it's very straightforward and obtains the full equation.

And again, what is very interesting is that we will obtain-- you look at this term. So this term is very complicated. It's essentially stochastic right? Because It looks very like stochastic. And because-- remember that this is the standard, normal variable. And all of them are independent.

So in principle, we have the sum of N independent normal variables squared. And it jumps out. [INAUDIBLE] a very beautiful result, and I recommend everybody also [INAUDIBLE]. I try to show you right now on the blackboard that if you sum up all this epsilon squared, that the limit when N goes to infinity, this term becomes deterministic.

So let me just show you basically what exactly is meant. So what I mean by deterministic is that of course, if I have epsilon squared, and then it's [INAUDIBLE] distribution, right? It's distributed between 0 and infinity, right? So this is [INAUDIBLE] function.

But my claim is that once I start adding more and more numbers-- and so on, and so on-- then this function will become more and more and more narrow. So it behaves like a deterministic, random-- like a complete deterministic variable in the large end limit.

And to do this, let me just write a very simple [INAUDIBLE] of what I mean. So essentially, remember that we have the sum of variables. Right? And for us to show that it's become deterministic, we need to show that it's squared.

This is [INAUDIBLE] distribution, which I defined as-- let's say you have a variable, right? And if I defined the distortion in the following way, now I defined [INAUDIBLE] distortion for this random variable which is equal to the sum of epsilon squared.

So if I write it here, then it turns out that each term in this equation is proportional to N squared, which is natural. But it turns out that the difference in [INAUDIBLE] limit is proportional only to N. Therefore, if you have this variable, which sum up more and more terms, then we'll have a variable. We have a distribution for this variable, which is moving in this direction.

And of course it moves this direction, but it becomes more and more and more narrow, basically. So as the limit of N turns to infinity, it becomes a deterministic. So I'd recommend everybody at home just do this very simple exercise. And you will see that essentially, the sum behaves as a deterministic quantity.

So just to do this, you need to you need to know the very simple properties of the standard normal distribution. First of all, the average expected value of epsilon is equal to 1, right? [INAUDIBLE] normal variable. And also, you need to know that the fourth moment of the normal variable is equal to 3.

So if you have this, then you can calculate this, which is trivial to calculate. And then you can come to this property that's once again probability distribution function, [INAUDIBLE] deterministic. It essentially becomes like a delta function.

So this is a very interesting result, because it basically explains why in the Black-Scholes equation, we have this, which is very weird by deterministic terms. And that's why the option pricing is possible. Because if you stopped pricing options-- like if you don't know anything about Black-Scholes, my [INAUDIBLE] no price for the option, because it [INAUDIBLE] although you do [INAUDIBLE], you still cannot eliminate your randomness completely. [INAUDIBLE] helps you with just too narrow the distribution of your outcome, but we're just not guaranteed at all.

So it's really very-- Ito's Lemma, which is usually inevitable [INAUDIBLE] probably like the first equation every written, basically is given without any proof. But this-- in reality, it's a very interesting limit. So it can be realized on the-- if you have two different time scales.

So a small time scale, which is dt prime is-- in the business sense, it responds to your [INAUDIBLE] frequency. It's when you balance your [INAUDIBLE] portfolio. And at the time dT, there's a time scale dT which is much bigger than dT prime. It's at the time at which you look at your portfolio.

So only in this very [INAUDIBLE] limit, when dT prime or dT prime goes to infinity, you strictly have Ito's Lemma. So actually, if you look even like is most famous

[INAUDIBLE] derivatives. If you look at this addition, you will see actually that the proof actually isn't correct. So just look at it and find what's wrong there.

AUDIENCE: [INAUDIBLE] normal?

DENIS GOROKHOV: Sorry?

AUDIENCE: That's what it is?

DENIS GOROKHOV: Yeah, this is what [INAUDIBLE] means. So if you use these two results here, you will see that your it's proportional only to N, not to N squared. So that's why your distribution becomes more than one [INAUDIBLE]. Because when you sum up, what it means is you sum up more and more variables.

Each of them was like random normal variables. So the average-- average goes like N. But the distortion-- the distortion, right? That's the standard deviation, right? You have it as square root of N. That's why basically, square root of N over N is small. So by increasing N, basically you become more and more and more deterministic.

So that's the main fact beyond Ito's Lemma. So that's [INAUDIBLE]. So I recommend everybody just look in detail, because this is the cornerstone of derivatives [INAUDIBLE] but as many books as [INAUDIBLE], not able to [INAUDIBLE]. So when I was studying, it was like I couldn't understand for a while. So it took me a time just to understand.

What else? And a very interesting thing now is that remember that we used Ito's Lemma-- and basically, we are able to obtain this equation. And this question very well known in [INAUDIBLE]. It's very similar to the heat equation. And heat equation can be solved using standard methods.

And I don't want [INAUDIBLE] here, it's straightforward. It's cumbersome but straightforward. And so if your option at maturity is given by some function, which is not really important here. Because you can write a very general solution. So what is here is essentially Green's function of this equation.

And this Green's function, if you look at this equation, is very similar to the probability distribution function, which we have on this slide in the very beginning. So this function is identical to this function, and the only difference is that the drift of stock [INAUDIBLE] disappears. And we are left only with the interest rate.

And so this equation, which is again, also very important for the derivatives pricing, is how we come up with the whole idea of Monte Carlo simulation. So this is nothing else as a Green's function, which basically tells us how the stock evolves in the [INAUDIBLE] neutral space. This neutral space is essentially some kind of imaginary world, like [INAUDIBLE] a world, where all the assets drift is just interest rate and not the actual drift. So it's very fundamental.

So it's very important things that the drift is [INAUDIBLE] drops out of all the equations. So the only parameter [INAUDIBLE] for option pricing is volatility. So this parameter's relatively easier to understand, right? Because that's how much money your deterministic investment basically makes.

So [INAUDIBLE] is the [INAUDIBLE] parameter. So naively, you could expect I need both mu-- let me just remind you what mu is. Mu and sigma, 20 minute parameter. But [INAUDIBLE] companies drops out of the picture. And this is because of the dynamic [INAUDIBLE], because it hedged the position.

And so now this equation-- since this is basically Green's function, and Green's function tells us what's the probability density of the stock at some time in the future, if the stock [INAUDIBLE] at some point initially, then basically this means that we can simulate their stock dynamics. And we can press derivatives, like usually a very simple framework.

So what do we do? We simply write the equation for the stock in [INAUDIBLE] world. Remember, the difference is that instead of the actual drift of the stock, mu, we substitute here by the interest rate. And this is basically how much money, roughly speaking, the bank account makes.

And what you do-- you start from some stock value at time 0, and then we simulate stock along different paths. So there are like three paths here. There could be like thousands. So now-- and you know, now, let's say we know the stock [INAUDIBLE] maturity.

And what you do-- the prices derivative is very simple. Essentially, you take the average of this payoff, over distribution. And you know distribution, because you just simulated [INAUDIBLE] price. And you just discount it with the interest rate.

So it's extremely simple. So principle, implementing this-- I'd say if you have [INAUDIBLE], it probably takes like maybe one hour at most, implement let's say price-- you know, Black-Scholes formula via a Monte Carlo simulation. So maybe if you have time, you can try this and see how your Monte Carlo solution converges to the exact result which was first obtained by Black-Scholes [INAUDIBLE].

So basically, this is a super powerful framework, which basically tells us something like this. So it's not applicable just to the stock prices, but it's also applicable to interest rate derivatives, [INAUDIBLE] derivatives, and foreign exchange derivatives, so on and so forth. Basically, ideas like this.

You have some [INAUDIBLE] derivative, depends on various financial variables. And you simply simulate all of them in the risk neutral world. Right? So you simulate all of them, and then you could [INAUDIBLE] payoff. And you just discount it. And that's how you compress derivatives.

So in principle, if you have a flexible IT infrastructure, like a financial institution, so you can implement it. And then you compress pretty much everything. That's basically how exotic derivatives are priced whose prices are not [INAUDIBLE] using analytical methods. And [INAUDIBLE] as a case for [INAUDIBLE] amount of derivatives.

So this is the whole idea, right? So Monte Carlo simulation is a very fundamental concept. So with [INAUDIBLE] world, and there are certain rules how to write these equations for different asset classes-- could be stock again, could be foreign exchange, could be credit, could be rates, whatever. And then you do some kind of something-- you find a average, and then basically you are done.

So this is how it works with the stock, and let me just explain how to generalize all these ideas for the case of interest rates create derivatives. So and let me just start from the very basics of the interest rate derivatives. So of course the whole point of derivatives is to allow financial institutions or individuals to manage their interest rate risk better.

So businesses need money to run their business. So big institutions, big corporations, have been [INAUDIBLE] in hundreds of billions of dollars of debt, and they know how to risk manage it [INAUDIBLE] efficiently. And just to make money, and not even necessarily financial institutions.

So of course if you borrow money, then you need to pay some interest. So you can think about interest of derivatives as some kind of option on the stochastic interest, because they'll say today, you can borrow money at 5%. But tomorrow, this rate can change.

So in order to control this uncertainty, you need to be able to just buy some derivatives, just to hedge your exposure, for example. Or it might just speculate. Maybe you just have some user traits that will go up or down. So it depends on the type of investor or speculator, whatever.

And so I mean this is a very simple concept of present value of money. If I have dollar today, it's definitely better than the dollar one year from now. Let's say I have a dollar, right? But I will get it only in one year from now. So how much does it cost? Clearly, [INAUDIBLE] interest rate at 5%, it roughly costs $0.95. Right?

Because what do I do? If the interest is 5%, then I'd take $0.95, and I'd put it into bank account, and I'd make 5%. So I will get like $1 in one year from now. So there exists very important cost of the present value. Or like [INAUDIBLE] value of money. Depending on where in the future you are, how much money it costs today.

And people talk about-- it's very often a fundamental notion of the fixed income derivative is the discount factor. So it essentially tell you that OK, if you have one dollar today, it's worth one dollar. But if you have one dollar in the future, basically it costs something else. So this a very important notion in finance.

So I'll tell a little bit more how [INAUDIBLE] them together, this functional [INAUDIBLE]. So another very important thing in the interest of derivatives right is a [INAUDIBLE] trait. So remember, [INAUDIBLE]. So we have discount factor. And the very important thing about discount factor is it should start at 1. Because a dollar today is a dollar. There is no [INAUDIBLE], right?

The [INAUDIBLE] should be [INAUDIBLE] or at least nonincreasing this time. So that's why it's very convenient to parametrize this kind of function. We use forward rates. So this is some positive, forward rates.

And [INAUDIBLE] very convenient. And remember, let's say, in the example below, like on this page, if all maturities earn 5%, then this is simply 5% a year. So for this example, basically your forward rate is just flat.

OK, so if this is an example-- and when you talk about interest [INAUDIBLE] derivatives, it's very convenient to model the dynamics of the forward rates. So again, it's very different from the stock, because it's got an additional dimension.

So if you model the stock dynamics, it's just a point process. Right? Let's say it's $100 today, and then you start modeling. Next, they'll go to $95, could go to $105, so on and so forth. But interest rates, it's more about curve.

So it's got an extra dimension-- it's the one-dimensional object. And the reason is very simple. In general, let's say if you borrow money for one year, then let's say you pay one percent. But if you borrow for two years, it might be that you borrow it for 2%, and so on. So there's a concept of the yield curve. And here basically tells us how much different maturities make.

So in a typical situation, resourced [INAUDIBLE]. If you don't have some [INAUDIBLE] of your [INAUDIBLE], which sometimes happens, it's usually upward sloping. This basically means if you borrow money for longer term, you pay higher interest. You can see it very easily. Like for those who have mortgages right there, it's always like 15-year mortgage rate is lower than 30-year mortgage rate.

And just here [INAUDIBLE] show-- to give you a [INAUDIBLE] up, or where we are right now in terms of interest rates, basically I just show you the yield of a 10-year US Treasury note. So what is 10-year Treasury note? Basically, the US government borrows money to finance its activities.

And then it works like this. Let's say I'm an investor. I'm giving the US government $100. And then every year, like for the next 10 years-- more exactly, like twice a year-- let's say they are paying me some coupon. Let's say if the interest rate per year is 5%, this means that if I give the US government $100, then the government pays me $2.50 every half a year. And at the very end, in 10 years from now, they must return $100 [INAUDIBLE].

And basically, if you look again how stochastic rates are right and what kind of amount we are [INAUDIBLE] right now, you can see that it always lasts about 50 years-- [INAUDIBLE] very distant future. From about '60s to about '80, '82, we can see a tremendous increase in interest rates. And this is something which looks very unbelievable right now.

So this problem nowadays. If one takes, let's say a mortgage, now a 30-year mortgage is maybe 4%, 4.5% nowadays. But let's say here, like 30 years ago, it was like a [INAUDIBLE] interest rate-- very high inflation. And mortgage rates were in double digits. It was not uncommon to pay like 15% if you will take mortgage somewhere here.

So the rates were increasing. But since then, we live in a very different environment, when interest rates gradually go and go down. So essentially, here, basically it shows in 1980, the US Government would pay 12% a year each year to borrow money for 10 years. So at the end of 2012, it paid less than 2%-- just 1.7%.

So [INAUDIBLE] like vertically [INAUDIBLE], you know? Something's going down. So in recent years, there is some kind of uptick here. But you know, we always get some kind of situation here. So where are we going? Nobody knows. But really, we're in this situation where interest rates are extremely low.

It was nothing like this, basically for the last 50 years. So it's very unusual, and you have these very low interest rates. This means that the economy is very weak, because this means there's not much demand on borrowing, right? Because corporations, like individuals, they don't want to borrow a lot, because once [INAUDIBLE] again, like supply-demand, right? Because if you want to borrow, basically you're willing to pay higher rate.

So also, of course another reason for this is because-- we live in a very unusual environment, because the government intervenes a lot on the market. So they're trying to make the rates as low as possible, just to make [INAUDIBLE] interest rates burden for corporations for, perhaps [INAUDIBLE], as small as possible. And hopefully, we'll go out of this recession. But as I said, this is very singular, very unusual environment-- just to understand what's going on.

And there a whole [INAUDIBLE] interest rate-- yes?

AUDIENCE: But it pays [INAUDIBLE] a non-productive access, like real estate, which are expected to rise with time, without, for example, [INAUDIBLE]. Doesn't it skew whatever investment is made toward access routes which are expected to rise with time? It may not be productive access--

DENIS GOROKHOV: Yes, yes, but right now, I mean I think even right now, lots of people are just scared to buy real estate. You never know what's going on, right? Because prices are still pretty high, so who knows what will happen?

So you're right. There is some kind of psychology [INAUDIBLE]. But many people who bought like 2006, whatever-- like before, they basically lost tons of money. You never know. So it's like when you buy some assets, you've got some finance. Let's say fixed rate finance. So you know how much you're going to pay, but what is it going to [INAUDIBLE]?

I mean, long term, it goes up, of course, but long-term basically means tens of years. But if you look like it's a real estate price, it's like for the last seven years. We are going up right now, but still, we didn't go through for the minimum. Like the [INAUDIBLE] maximum, which you had before, basically. So you never know.

Yes, and so there's a whole world of interest rate derivatives. So I'm just very briefly explain what it all means. So usually, here I mentioned it's all about Treasury. So it's all like government-- it's kind of yield implied from the government bonds. But usually, all the derivatives are linked to another.

A very famous rate is called LIBOR. And LIBOR-- roughly speaking, it's a short-term rate at which financial institutions in London borrow money from each other on an unsecured basis. So there's a lot of caveats here on this definition, but that's roughly what that means. And there is like a fundamental derivative [INAUDIBLE] LIBOR swap.

So the standard [INAUDIBLE] LIBOR swap is something like this, basically. It's paying once in three months. It's paying three months' LIBOR rate. And so this is stochastic, right? So basically, every day, there is this certain procedure, which tells us what this LIBOR, this short-term borrowing rate is. And in exchange for this, if you're paying out this LIBOR swap, this LIBOR rate, you are receiving the fixed rate, which is diminished.

So this is like fundamental [INAUDIBLE] basically. It's like especially, if you believe that rates will go up and you just want to speculate, basically you're trying to [INAUDIBLE] long LIBOR and short fixed rate, and vice versa. So this is a very important instrument for pricing.

And it's all kinds of derivatives linked to this LIBOR rate. For example, you can talk about a swaption. What is a swaption? Selection is a derivative to enter an interest rate up in the future. Remember like in the equity option work, let's say if I have a call option on the stock, that's the right to buy a stock at a fixed price-- it's fixed today-- like at sometime in the future.

Here, this is basically the same idea. If you're here today, at sometime in the future you can enter a swap, a kind of contract, which pays various lengths and there is some price given for today. And there are also all kinds of false derivatives.

You can talk about rates. Basically you can buy or sell options on a particular LIBOR rate. Or there's also cancel-able swaps, which basically are you can enter a swap, but if you don't want to pay, like, let's say, high rate anymore, you can cancel it. Of course, it's affecting the price so on and so forth.

So, very important idea if you think about all these that it turns out that when you price all these derivatives. Their price depends on these discount factors. And the discount factors depend on these forward rate, which is basically our primitivization. But it's very important, very convenient, to work with these forward rates.

And when we model interest rate derivatives, use the Monte Carlo simulations. And there are no analytical models available than if models of dynamics of forward rates. And you can ask a question. So how can we get, basically, this curve in practice, or this curve?

And it turns out that the swap market tells us how to obtain this curve. So here I show some quotes, real market quotes, for interest rate slope of different maturities. Let's say two years, three years, four years, and so on and so forth. And then if you add this number and this number, then you obtain the swap rate.

So if you take these swap rates, then it shows that you can show very easily that if you know all these numbers, then you will be able to obtain this curve in a pretty unique way. So because of this market of swaps-- so once again, if you add these two numbers here, then basically it tells you that, for example, for this instrument, we'll say, five years.

For the next five years, I'm going to pay roughly like 0.75% a year. Right. So these two payments, basically, [INAUDIBLE] 0.75% in exchange for the LIBOR payments, right? So if I enter a swap-- so I know that the I will be paying fixed-- but obviously some floating, which is random, because we don't know what it is.

And my [INAUDIBLE] is a pretty complicated concept. That is very simple. So basically the swap market allows you to obtain this discount factor-- basically this function-- which tells you how much your dollar is the future is today. So if you know how much a dollar is, then you know how much C dollars, basically, cost.

Then basically, let's say you have C dollars. Then you simply multiply them by the discount factor, and then what the present value of your fixed rate payment is. So remember that finals also become very important things. And finals [INAUDIBLE] is a derivative world. What is called PV or present value of all our future payments, right?

So we have some future liability, which is something very complicated. I say, I'll pay you something very complicated, pay off in 10 years from now. But we are trying to understand how much it works today.

Because idea for this business is clients come to the bank. And they say, I want this derivative. You sell this derivative. You charge the money right now, and you spend this money on hedging. Of course, you try to charge them a little bit more because you need to still make living. But in [INAUDIBLE] basically is like you've spent most of your money on hedging.

But you to try to come up with a number today. Here's, again, a very simple example. So if you know, once again, how much your dollar is in the future, then you can present value PV every payment. So let's say in 10 years from now, g is equal to 0.5, then if you pay $1,000 the present value is equal to $500.

Because, again, the argument is very simple, right? You take $500 today and invest for 10 years, and you get $1,000 in the future. This is the duplication argument. And that's a very important thing here, is that if you have an interest rate swap, which is paying LIBOR. And let's say on a notional.

Let's say I pay your LIBOR, which is some rate which is [INAUDIBLE] percent. LIBOR is like a 1% a year, for example. Then notional of the slope is $1 million which means that the [INAUDIBLE] rate payment is based on $1 million times 1% is $10,000.

So it turns out that very interesting thing is that if you pay LIBOR rate and if you pay the notional at the very end, then the present value of this is equal to the notional. So it's the beauty of floating rate is security. Elaborate excuse is basically that if you pay the current market rate all the time, then the price of your security is always equal to the notional.

It's very nice fact which is also from the mantel here. And very interesting thing would happen after [INAUDIBLE]. All the derivatives could become collateralized. So you need to post the money all the time. So this is not the cost of OIS discounting, which I don't talk about here.

The main idea which you need to understand here is that we have this function, like discount function, which shows us again how much the dollar is worth in the future. And using this function, we can press all kinds of swaps. So we can PV the value of the swap today using this.

So the idea of interest rate derivatives it's all about dynamics of the yield curve. It's basically how your discount function or how your yields, future yields, evolve. The whole idea is similar to the stock. So again, at time 0 you start from some curve. For example, something like this, right? From some curve which is shown here.

And then it stopped evolving and you want to be able to model it mathematically and press all kinds of derivatives. So there is like a very interesting difference between stock options and [INAUDIBLE] options because for the stock options, we know the price today. If it's a liquid stock, it's just [INAUDIBLE]. We know what it's trading right now.

But for the yield curve, it's different. We first need to take the swap markets quotes and do what is called bootstrapping to get the function d of T. The next step, we need to specify the volatility of different forward rates in the future and we need to come up with some dynamics which describes the future dynamics of forward rates. And then once we have this, we can use the Monte Carlo framework to press all kinds of derivatives.

So before I start talking about the HJM framework here, I just want to mention that there are some other more simple models which are historically appear before the HJM model which basically describe the dynamics of the short rate.

And so the most famous ones are the Ho-Lee model, Hull-White model, and so-called CIR model. And basically, the idea is that if you have this function for forward rates-- which I [INAUDIBLE] here. So they describe dynamics, instantaneous dynamics, of this rate. So instead of modeling the whole curve, you model only just this short rate and so on.

So some of these models. I particular [INAUDIBLE] of the HJM model. Some of them, I'm not. But just to mention. And basically the idea, then, of the interest rate derivatives, for example, let's say I want to price an option that in five years from now, I enter a particular interest rate swap which pays 5% on the fixed leg and receives LIBOR.

So I need to model the dynamic of future yields. And remember, it's a very important thing that, again, because we have the curve, now we have two different times here. For the stock derivatives, we just basically write dynamics. G of ST is equal to something. And t is just basically instantaneous time.

Here t stands for instantaneous time. And T, capital T, stands for the future time. Here. So essentially if you're here, you're looking at the forward rate somewhere here. And then you basically describe with dynamics.

I don't want to go into details, but again, using this very fundamental result [INAUDIBLE] pricing like Ito's Lemma. You can derive the equation for this drift. So the problem is it turns out it's always the case in the Monte Carlo simulation.

So you [INAUDIBLE] some time equation and you have drift and you have volatility. So it turns out that this drift, the real time drift, because you hedge drops out of your equation. And it turns out that for the interest rate, there is some complication in the risk neutral world. This real world drift [INAUDIBLE] by some equation which depends on sigma.

So if you do the calculation, then you will see that in the risk neutral world, if you [INAUDIBLE] a distance of [INAUDIBLE], which is some non-local equation. But it is what it is. So it's very straightforward. I encourage you just to, if you have time, to go through this and really understand how it works.

But now once we have this, the model for interest rate derivatives is very simple. And remember that in the stock world-- let me go back just to this equation. So we started from some stochastic differential equation. And then we simulate different paths. And then basically we average over the pay-off here at maturity of the derivative, when actually we do the payment.

And here it's very similar. So we have some initial curve which we obtain from the market today. And this curve dynamics is described by this equation. Then we have distribution of this curve in the future, and then you can press all kinds of derivatives.

So again, it's a very fundamental framework. So very general. So once the curve and the volatility are known, you simply round this simulation and you get your pay-off. So basically that's how it works. And now an like example, which is basically of this HJM model. It is basically credit derivatives.

So I don't have much time, but just mention-- I'll go very briefly what's going on. So if you give money just to someone, like to the corporations, then there is a probability that you won't get your money back. So corporations issue bonds, financial instruments to raise capital. It's, again, very similar to the US treasuries.

And so you give them $100 and they pay you 5% percent every year. And then let's say in 10 years, if it's a 10 year bond, they are supposed to give your money back. But this might not happen. Corporations default because they make their own decisions. Like something went wrong with economy, and so on and so forth. It happens.

So there is some risk which is indicated here. We just call it default risk. So corporations or private individuals, they have a right to default. So they can default. And this is reflected in the coupons which they pay.

So for the US government in 2012. A 10 year bond would pay just 1.7% a year. Again, they are in extremely low environment which looks like almost nothing. And remember that even if you're investor and if you buy this bond, then you get your 1% interest but then you need to pay taxes on the profit. So the return is really very small.

So then, of course, if you're an investor, then OK. The US government securities are assumed to be risk-free, so you won't be able to lose money. So this is a very important benchmark. But then you can buy bonds of corporations. But, of course, to compensate for possible default, they pay higher coupon.

For example, as of 2012, Morgan Stanley bonds would pay around, let's say 5% a year. Significantly higher. Some government are now very close to default. So some time ago, for example, the Morgan Stanley bonds would pay 5% a year. But say, Greece bonds would pay 25%, 30% a year.

Because vis a vis no else was going to happen there. It's clear that the economy is not in good shape and it all depends on the bailouts. Or these bailouts are conditioned, for example, that the right government-- if you'll be in power and the [INAUDIBLE] is unclear. So there's lots of uncertainty.

Such uncertainties, that's why [INAUDIBLE] are yield. Investors tell you would require very high yield. And the great derivatives, as a fundamental instrument, is default swap. So if you have a risky bond, then to protect from default you can go, let's say to a bank, and buy great default swap. It basically means that if you hold a bond and default happens, then the seller of this protection will compensate you for the loss.

For example, let's say you bought a bond at $100. And then, let's say, in one year the corporation defaults. And then what happens in this event? Then court. Court happens. And the judge decides how much money is recovered.

And this money is distributed to the bond investors. They're first in the queue. And then if, let's say, $0.70 on the dollar is recovered, then the default swap will pay you $32 vis a vis your loss.

And very fundamental concept in the world of credit derivatives is market implied survival probability. So in principle, credit default swaps are available for different entities. Let's say like Morgan Stanley. It could be Verizon. Could be AT&T and so on and so forth.

And [INAUDIBLE] require different payments, for example. Let's say if credit defaults off [INAUDIBLE], properties like [INAUDIBLE] maturity will pay around 100 basis points. And if there is some Greece, probably, you pay like 500, maybe 1,000 basis points on some like this. So market differentiates.

And based on this, you can then do a very simple calculation. And you consider, it's very easy to come with a concept of the survival probability. Roughly speaking if, let's say, default protection on some reference entity is worth 1% a year. And then what do we see?

Then with probability 99% a year, you will get your money. If probability 1% per year, you will get nothing. So you can think about it like this. This means you can say the probability to default is roughly 1% a year, in this case.

And then we could talk about survival probabilities, which is basically one [INAUDIBLE] for probability. And you can then come up with the concept of survival probabilities, which you can again parametrize these forward rates which are called hazard rates.

So credit derivatives in a sense, they similar to interest rate derivatives. Remember, in the case of interest rate derivatives, we were talking about discount factors. So this is like the present value. Present value of money.

In terms of world of credit derivates-- besides this, because of course interest rates are also very important for derivates-- we talk about survival probability. Today it's equal to 1, but then it decays. And let's say if you have a US government, it always stay at one. And let's say if it's like Morgan Stanley, it goes like this. If it's some distressed European [INAUDIBLE], it will go like this.

So basically it's market probability of default based on the credit default swap market. And the idea of the HJM model for the credit derivatives is that similar to the dynamic of forward rates. In interest rate case, you simply describe the dynamics of hazard rates which parametrize your survival probabilities.

And now let me see. Let me show an example of very important type of derivatives, which are priced using credit models. Let's talk about the corporate global ones. So it's a very simple instrument.

Again, I'm a corporation. I borrow $100 from you. And let's say I'm paying you 5% every year. But I have the right at any time-- or, let's say, once in three months-- return you this $100, which could close the deal.

So why is that so valuable for the corporations? Because today's environment [INAUDIBLE] I borrow at a very high rate. In this example, let's say I am paying 5% a year. And I entered a 10 year bond and there's $100 million notional.

So basically this means that every year, I am paying to the investor 5%. $5 million. But let's say I'm paying 5%. I need this money to run my business and so on. So it's some burden, but usually all the corporations have significant amount of debt. So it's good to have debt if you know how to manage it.

Now let's say in three years from now, situation changed. So now I can borrow money for seven years, because initially I issued the bond for 10 years. And now I have seven years remaining, but it turns out I can issue just a 3%.

Basically this means if I do this, if I exercise my call option, then I will save 5 minus 3-- 2%-- times $100 million times seven years. So it's only for $5 million. So that's kind of why callable debt, it's good to issue because you can save money.

It's very similar to what's happening right now also for private individuals. Because in recent year or couple of years, there was lot of refinancing activity in the US. Remember rates are at historical low right now. So rates are going down, down, down.

So let's say if you took out a mortgage here at 6%, it was like you would refinance at here, for example, the same mortgage. You could get like a 3.5%. So the same [INAUDIBLE] has happened to corporations. So in the US, by default, all mortgages are callable. And basically by default, everybody has a right to refinance.

So it's not like you issue a 30 year bond and then even you're paying a huge coupon, even you can finance lower percent-- which might be the case for corporation, by the way. But by law in the US, all the mortgages can be refinanced.

So basically, that's the idea. So if you price the cost of instrument as a callable bond then you need to take into account, of course, the interest rate risk because you need to understand what is the current level of interest rate you can charge. And also you need to take into account the quality of the issuer.

So if, let's say again, Greece. Or, let's say, Morgan Stanley issue debt right now, then Morgan Stanley would pay significantly less. It's all [INAUDIBLE] on the fair market. Basis result and subsidies. And, of course, Morgan Stanley would pay significantly less in the interest because for the case of Greece, it is a much higher default risk.

And as I mentioned, the idea is that you, in the world of credit derivatives, there is the concept of hazard rates which you gain some curve which shows how risky the issuer is at some point in the future. And here I show the dynamics for the forward rates, and here is the dynamics of hazard rates.

It shows you, basically, how risky the issuer is. And then using similar approach-- I show an exercise-- you can prove again-- it turns out if you know the volatility of hazard rates, then you know how to simulate the dynamics of hazard rates.

So essentially, it's the dynamics of all this. So again, it's the idea-- let me go back just to the stock case-- again, it's the idea, it's very simple. So you have all the dynamic variables like rates and [INAUDIBLE], in this case.

Then what you do, you simulate that in risk neutral world. You have different paths. And then you simply average over the pay-off. So this is the beauty of the risk neutral pricing. There is a visual framework which is basically implemented at all the major banks. And we just really like the right approach to price very [INAUDIBLE] derivatives for which it's very hard to find the exact analytical formulas.

And let me show you one example of securities, which are issued by big banks. And that's where this HJM model and Monte Carlo simulation are used all the time because the pay-offs are very complicated. And example of such a product is called structured note.

So what's a structured note? It's, again, corporations need to raise money just to run this business. But, of course, I cannot just get this money for free. I need to pay some interest. And again, if you look at what happened last year.

Again, at the end of last year, let's say a US 20 year bond would pay 1.7%. And if you also pay all the taxes, then you probably get something like 1.1%. And this might be even lower than inflation. So investors, especially long-term investors, they are not interested in investing in the US treasuries because although it's risk free, but there's no return. So you want to generate some money.

So what can you do, then? OK, so you don't want to [INAUDIBLE] to treasuries. So then you can try to find some corporate bonds. Again, corporates are risky compared to the United States government. So typical coupon paid by the corporate bonds would be higher. So let's say 5% for a non-distressed typical US corporation.

But again, 5%. Then you need to pay, let's say, 30% tax top of this. So you're left 3.3%. There's inflation and so on and so forth. So it still looks like a low return. Of course, [INAUDIBLE] below, you can buy some distressed bonds, say from Greece or maybe from some distressed corporations, which is a much higher.

But it becomes more like gambling. There's so much [INAUDIBLE] so it's more like you can get very high return, but you can lose everything because basically you're bearing very high credit risk. So what to do in this situation?

Turns out that banks issue very special securities called structured notes which are very attractive to some investors. So let's say Morgan Stanley-- but instead of issuing vanilla bond, I am issuing-- and at 5%, let's say for 10 years-- I issue a bond which pays 10% a year. So much higher coupon.

But I pay you 10% only if certain market conditions are satisfied. So let's say market condition like this. 30 year swap rate is higher than two year swap rate. Let's go back to the picture which I drew. So essentially this means that if you borrow money, then the short term borrowing rate is smaller than the long term borrowing rate, which usually is the case.

So basically, let's assume I pay you 10% percent if two conditions are satisfied. 1% is a 30 year borrowing rate in the economy right now. It's higher than two year borrowing rate, which is this condition. Plus this second condition. S&P 500 index is higher than 880.

So now if these conditions are satisfied, then the investor will get 10%. If one of these conditions breaks down, the investor would get nothing. So there are many investors who would like to bare this kind of risk because they have certain view on how the economy would develop.

Because right now, for example, S&P 500 index is pretty close to 2,000. So it's very unlikely that it'll go down by the factor of two, which is 880. So it's very low probability. And then also investor believes that this will never happen. So we always will be in the economy where it's still more expensive to borrow long-term than short term.

So in this case, it turns out that the coupon can be enhanced. This is a whole idea of the structured note. So instead of setting like a plain coupon, 5%, I am selling [INAUDIBLE] the derivative. And even investors like, it's just gambling but in educated way because there's certain economic meaning of these conditions. But this can get high return.

And this is a very popular way of financing because it turns out that investors' brains are kind of instruments, but they are very unique. There's a lot very liquid. Therefore when issued this instrument, even if you price it correctly using all the models, the bank or financial institution which uses these instruments can make some extra money. So effectively it's cheaper o issue these instruments than to issue vanilla bonds.

And all of these big banks, they have all the [INAUDIBLE] of this [INAUDIBLE] derivatives. So they know what they are doing. So this kind of product, and they're hedging their exposure. And they realize some profit because you can't identify how much [INAUDIBLE] instrument is. So it's good for banks.

And it's also good for investors because they are looking for this kind of yield enhancement. They want to [INAUDIBLE] a higher yield. And they are taking [INAUDIBLE] into to take this risk. But again, it's an educated risk because like, this condition, for example, here, they have a very clear economical meaning. So if an investor understands what's going on, then it's a reasonable risk.

And, of course, what do you do in this case if you want to model something like this? Then it's very complicated to find any kind of analytical summations here in the real world. So what do we do?

We simulate the stock market price. We simulate the 30 year yield and 10 year yield. And we simulate Morgan Stanley's [INAUDIBLE]. And we do it all simultaneously, at the same time. And then we see in the Monte Carlo simulation if this condition is satisfied for every coupon [INAUDIBLE], then we're paying 10%.

If something is broken, then we are paying 0. So if we simulate money [INAUDIBLE] like this and then we calculate the average value of it. And then we come up with the price and then we call this price to the investor. And again, I say, these products are very nonstandard. That's fine. You can make some extra money.

And as a firm, you save money because it's cheaper than to issue plain vanilla bonds. And just to give you the idea where we are in terms of numbers. So here there is a graph of difference between 30 year borrowing rate and two year borrowing rate for the last decade.

So you see, this difference always positive. It was negative only very shortly for some time around 2005, 2006. So it's very interesting thing. So when you price derivative, then there's a notion of market applied numbers. It turns out if you look at how different instruments are price on the market, then the probability.

Then you can ask a question. Let's say if I round, for example, this stochastically for the last 10 years, then how would the probability that this difference is positive. And then [INAUDIBLE] probability is only 80% percent. Whereas in reality, it was realized only for a few days. So it's significantly lower.

So basically, then, the investor says like this. So market give me the discount, like 80%. But I know that this almost never happen in the past. Therefore I believe that it will not happen in the future. Maybe it will happen, but I will still make some extra money because of this. So basically we have [INAUDIBLE] by a factor 1 divided by 0.8. 1.25%.

Second thing is about S&P 500. If you look at the history of this index, which is basically the main US market index, then you see that it was historically above 880 level for 94 days out of 100 days. So very, very high probability.

But the market implies this will be the case only in 75% case. The credit investor would say like this. OK, now S&P 500 is around 1,800. So what the probability it's going to drop below 880? Of course there is some probability, but if it's going to happen because it will mean a very serious recession, and it looks like the economy is improving.

The market might drop down, but maybe to the level of 1,500, 1,400. But not that low. Therefore the investor believes that he, by taking this risk, he will again get a higher coupon. So [INAUDIBLE] very popular instruments which are [INAUDIBLE] the Monte Carlo simulation which can have big businesses, for example.

Like Morgan Stanley, whose goal is to raise capital by selling these exotic products and hedging them using the Monte Carlo framework. And if the interest rates are crucial for [INAUDIBLE], then we use the HJM model for simulating interest rates. So that's everything I wanted to tell you about today, so thank you very much.

[APPLAUSE]

DENIS GOROKHOV: Yeah?

AUDIENCE: [INAUDIBLE] simulation. Is there some choice-- you might make certain choices based on historical precedence?

DENIS GOROKHOV: It's a very good question. So, in the [INAUDIBLE]. So here's what happens. So let's go to a very simple case of stock prices. So again, r here basically is just the borrowing rate. It's like, let's say, whatever the bank account gives. So [INAUDIBLE].

So the only parameter known is volatility. So usually, you have liquid stocks, for example. Like IBM, Apple. Then there are a lot of derivatives traded, which are very liquid. This means that you can imply this sigma from the price of liquid derivatives.

Because you know, for example, that this particular option-- let's say today Apple traded at 600-- and you know that at the money option, like 600. And one year now, for example, it's worth whatever. Like $50, for example. By knowing this, you can imply this sigma.

So the whole idea is like this. So you take very liquid derivatives, like standard call options, and you imply this sigma. And then you use this model to price really truly exotic derivatives, which are not vitally available.

That's how big banks make money. Because we know how to price them. We have clients come in. And we see the prices of [INAUDIBLE] instruments and we buy them [INAUDIBLE]. So very often what we do is that we do some very complicated deal, but we have an ability to off-load it into simpler contracts, which we know how to price. That's the idea.

And the same is true for all the other derivatives, from credit derivatives or [INAUDIBLE] derivatives. So you try to imply the sigma from the market. If there is no way to do this-- which is very often the case for credit derivatives because for the credit derivatives, credit is not very liquid traded. Then the best thing that you can do is to take historical estimates. So we also do this. There is nothing else.

Yeah. Yeah?

AUDIENCE: On your last slide where you talked about the implied frequency of the S&P 500 being lower than 880?

DENIS GOROKHOV: Yeah.

AUDIENCE: Was that from historical quotes or current quotes?

DENIS GOROKHOV: OK This number, I think, if you go to the end of 2012 and go back to 2002. 10 years into the past. Then I think it was above 880 in 94% of case. We can go back.

So remember, just to the slide I showed in the very beginning. Here is right. So 880 is somewhere here. 2012 is here. You go back 2002. It was below 880 around 2000, internet bubble. And around, say, 2008, 2009 when we had major banking crisis. [INAUDIBLE] just now. So you can see probability is not very high based on historicals.

These kind of people believe that in the future it might happen, but then the stock will go back again because the government will intervene and so on and so forth. That's the way of thinking of these investors who invest into structured notes like this.

AUDIENCE: So for the implied frequency, that's from the current--

DENIS GOROKHOV: Exactly.

AUDIENCE: --option prices--

DENIS GOROKHOV: Exactly. Exactly. Exactly. Exactly. So now that's how historical was obtained. Ah, let me see. [INAUDIBLE]. Yeah. Well, let me see.

So, yes. So it's like this. So you're today and you have your Monte Carlo model. And you simulate going forward for 10 years and you see what the probability to be below 880. And actually, much higher because usually the market is extremely risk averse.

So if you're buying a [INAUDIBLE] of the money option you usually-- there is a [INAUDIBLE]. Because if this happens, if you don't really like [INAUDIBLE] enough money, basically that you're out of business. That is how I obtain this number, what, 75%. Whatever. OK. Yeah?

AUDIENCE: So is the pricing of these more exotic products totally reliant upon Monte Carlo, or are there other techniques?

DENIS GOROKHOV: I mean, usually it's Monte Carlo. So there are some derivatives for analytical approximations available. For example, for interest rate derivative. Swaps are like a very simple linear product. To price them, you need discount function. So it's just a matrix. Of course, it's all down. Just simple arithmetic.

For swaptions, standard swaptions, there is a model called SABR model which allows some kind of simulated [INAUDIBLE] solutions, which are approximate but of high quality. Then you can do it. But there are different schools of thought.

Because with some approximations, which might fail for some if maturity is very long, or it's very-- [INAUDIBLE] is your only option. So very often what traders do, even if their official numbers are only simplified models which [INAUDIBLE] some formula, they still round the Monte Carlo simulation for the whole portfolio to understand what the most complicated model, like in terms of your present value of your portfolio, in terms of the risk.

But, of course, this kind of double [INAUDIBLE], which is just short. It's impossible to build any meaningful [INAUDIBLE] model. You can do something, but you won't be able to be competitive. It's just all Monte Carlo simulation.

AUDIENCE: So you said usually this whole summation process takes an hour on a Mac Lab program?

DENIS GOROKHOV: No. No. It takes probably like one hour just to write the whole program, because it's very simple. So what you do, you have Brownian motion. But what you do. Mac Lab generates Brownian motion. So you just do it.

And then you write the change in your price is equal to your drift, which you know, plus some random number. And you basically just simulate different path. And then if you press a call option, you know the distribution of your stock prices, let's say, in one year from now with maturities. And you just do average.

So it might take someone 15 minutes to write this kind of program. This is so you can verify numerically the [INAUDIBLE] of the-- [INAUDIBLE] Black-Scholes formula, for example. But the idea fits very simple here.

But, of course, for these complicated models, [INAUDIBLE]. For [INAUDIBLE] structured process, like a HJM, because it's already one dimensional object. And, of course, it's much more complicated because besides pricing, you need to have this idea of collaboration as you mentioned, because these volatilities are not just usually historical. They implied from other instruments.

So what you do in practice, like this. So if you have liquid instruments, liquid options, you have the model. But the model has known parameters. First, we do the collaboration. So we make sure that our model prices all the simple instruments.

And then we take the derivative whose price is unknown because it's just something very complicated. And then we just price it, but our models create the simple derivatives. Then this model, after pricing it and running sensitivity for special market parameters, tells us how to hedge it. That's the idea.

AUDIENCE: You ought to do post-hoc analyses to see how the models did in the past so you can adjust them.

DENIS GOROKHOV: Yeah. Yeah.

AUDIENCE: Is that a big part of what you have to do?

DENIS GOROKHOV: I will say in general we are moving to this direction. In general, of course, for Monte Carlo-- from the [INAUDIBLE] point of view for complicated Monte Carlo model, it's very difficult to do technically. It's very difficult. But if you do it, you cannot afford simple models like for swaps and so on.

AUDIENCE: --historical experience with the projection that you made.

DENIS GOROKHOV: No. But the situation, it's very difficult. So we don't make any predictions here, remember. It's risk neutral pricing. Just no prediction here. What we do here is like this.

If we are a bank and we want to trade all kinds of very exotic derivatives which nobody knows how to price but we have clients who want to buy them with different reason. Might want to speculate or they want to manage their risk exposure and so on and so forth.

So nobody knows except for like 10, 20 banks, how to price them. Because this is like, you need to have infrastructure. You need to know how to do this. Then you need to have some business channels how to off-load this risk. So this is some very exotic products.

So now the idea of [INAUDIBLE] is like this. Remember in the case of Black-Scholes. You buy an option and then you hedge it by holding the [INAUDIBLE] for underlying. So you don't make any money. But you want to make sure that whatever happens to the market, you're fully hedged. So the market moves here, you don't make any money. The market moves down, you don't make any money.

So the way how you make money in this situation, basically this Black-Scholes formula, in this case, the price which you charge for the option is the price of the [INAUDIBLE] strategy. So if you charge a little bit more, this is extra money which you can make.

So it's very different. So what you just mentioned is like propriety business. Big banks say, I'm not supposed to do this. It's more like a hedge fund [INAUDIBLE]. Very different models.

What we do, we try to manage big portfolios of derivatives, all kinds of derivatives. And we try to price them and charge a little bit extra so that we can make our living. But on the other hand, we don't take any risk. That's the idea. [INAUDIBLE] just models.

So from point of view in terms of testing historically, you can still ask a question. Let's say if I go back 10 years. And let's say 10 years ago, I would sell, for example, this stock option. And for the next 10 years, using historical data, I see basically how my model then tells me what my Greeks or like what my sensitivity is to [INAUDIBLE] underlying. What my sensitivity underlying is.

And then you can ask a question. How was this delta H performing historically? This is a [INAUDIBLE] question because you assume that the model pretty much continues. But maybe if your dynamics is very jerky, then you can just lose money because you just don't take into account these effects.

Here's an example of [INAUDIBLE] which we [INAUDIBLE], but it has also to do with prediction here. So it's a whole different world. So it's risk neutral pricing. So we don't take any risk. That's the whole idea.

But due to the fact that derivatives are very complex, even in this case, still banks bear some residual risk, because remember we cannot exactly afford it, the risk. So we still have some assumptions that we can re-balance our position dynamically and move forward, basically, and not lose money. That's the idea of it.

AUDIENCE: I have a question about the Monte Carlo pricing. You can set up the Monte Carlo using implied parameters from current prices of various derivatives in the market, which gives you a good baseline price. I'm wondering what other Monte Carlos do you do to have a robust estimate of price and hedging cost.

I would think that there would be, I don't know, maybe some stress scenarios in the market or alternatives. You probably don't just do one Monte Carlo study with current parameters. You probably have different sets. And I'm wondering how extensive is that?

DENIS GOROKHOV: Absolutely. You are right. So if you just do the Monte Carlo, then you just know the price. But price is nothing, because dynamic hedging, all this business of derivatives, it's not just about how much it's right now, but what to do if the market behaves this way. So of course you could collate all your Greeks. That's very important.

But Greeks is like, say, your delta. It's all about linear in terms. So of course it's a very important thing. What happen to the portfolio, let's say, if there is a very sharp, for example, jump in interest rate. So let's say, what happens if rates jump forward by 1%. Or if they jump down.

What happens if volatility in a particular time, region, for example, blows up. You're on all those [INAUDIBLE]. So it's big departments at the banks who look at all this kind of risks. So it all comes to one business unit which looks all kinds of risks of the firm.

It's a very big thing for the bank. This notion of stress test. Basically right now, all of the banks are very heavily liquidated by the government. So the government can tell us what happens. For example, for the whole bank-- not just for a particular desk which trades for those swaptions. What happens to all your bank, to all kinds of cash flows which you can have if, let's say, interest rates jump by 100% percent.

We have a huge group of people, [INAUDIBLE], IT, risk managers, who are looking at all these numbers trying to understand it. And for a big bank, very non-trivial problem, actually. So it's very good point. But, of course, we do as good as we can. Yeah.

AUDIENCE: Well, thanks again. And for a little time afterwards for--

[APPLAUSE]

DENIS GOROKHOV: Thank you.