

2.003 Problem Set 6

Assigned: Fri. Mar. 11, 2005

Due: Fri. Mar. 18, 2005, in recitation

Problem 1 Archive Problem 14.2

Problem 2 Archive Problem 14.3

Do not solve for the transfer function. Rather, find a differential equation which describes the system in terms of an input $F(t)$ and an output $x(t)$.

Problem 3 Archive Problem 14.4

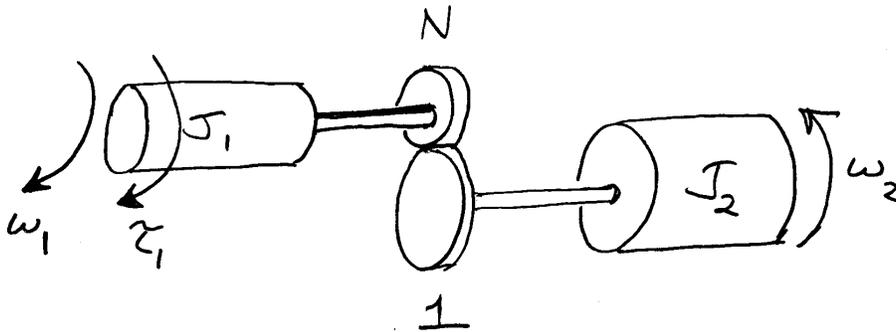
Problem 4 Archive Problem 14.5

Problem 5 Archive Problem 14.7

In section b), assume that the wheels have a radius R .

Problem 6 Archive Problem 14.8

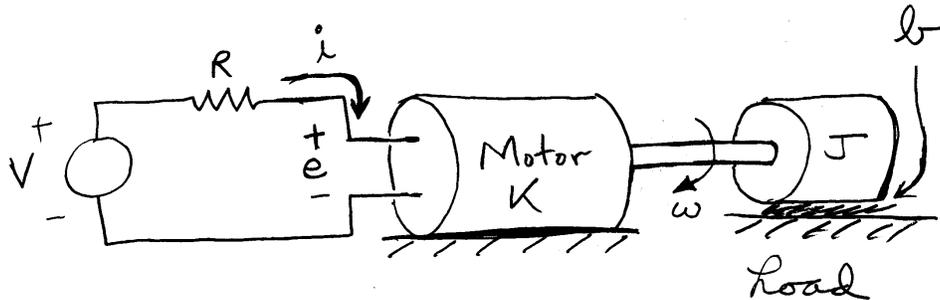
Problem 7 This problem considers the system shown below in which two rotational inertias J_1 and J_2 are connected by a gear train.



The rotational speed of J_1 is ω_1 and the rotational speed of J_2 is ω_2 . An input source of torque τ_1 is applied to J_1 in the direction of ω_1 . As shown in the figure, the gear train has a ratio of $N : 1$; that is, $\omega_2 = N\omega_1$.

- Assume that the input torque source has a constant value of $\tau = \tau_0$. What value of N will maximize the acceleration of the load $\dot{\omega}_2$?
- For this acceleration-optimum gear ratio, what is the equivalent inertia of J_2 as seen by J_1 looking through the gear train? That is, what is the reflected inertia of J_2 on the J_1 side? How does this compare with J_1 ?
- The power input to the system is $P_{in} = \tau_1\omega_1$. For the optimum gear ratio calculated above, make a plot of $P_{in}(t)$ assuming that the load starts at rest at $t = 0$.

Problem 8 This problem focuses on the motor connected to a load shown below. The motor is driven by an input voltage V in series with a coil resistance R . The motor is assumed ideal, with no energy storage or losses inside the motor. The motor is connected to a load inertia $J > 0$ and rotational damper $b \geq 0$. The motor shaft and load rotate at angular velocity ω . The motor applies a torque to the load $\tau = Ki$ in the direction of ω ; correspondingly, the back emf is $e = K\omega$.



- a) Write a differential equation describing the system in terms of the input voltage $V(t)$ and the output speed $\omega(t)$. Write an equivalent differential equation with input $V(t)$ and output $i(t)$.
- b) Assume that the system is initially at rest, and that at $t = 0$ the input voltage takes a step $V(t) = V_0 u_s(t)$. Solve for the resulting transient in $i(t)$ and $\omega(t)$, and make a plot of these two quantities as a function of time.
- c) What are the steady-state values of i and ω ? In steady-state, write an expression for the power being dissipated on the mechanical side in the load damper b , and on the electrical side in the resistor R . How much power is being supplied in steady-state by the voltage source? Is this in balance with the dissipation? In this steady-state, how much kinetic energy is stored in the load inertia J ?
- d) Make a plot of steady-state load power dissipation as a function of load damping b for $b \geq 0$. What value of b results in maximum power dissipation in the load? How does this compare with the electrical equivalent damping term K^2/R ? For this maximum power value of b , how much power is being dissipated in the resistor R ?
- e) Finally, suppose we allow negative values of the load damping. Note that a negative damper will *supply* power to the load. For what range of $b < 0$ will the system be stable? For what range of $b < 0$ will the system be unstable?