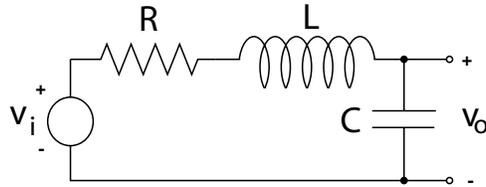
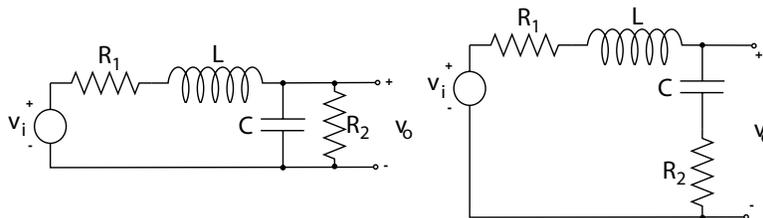


Problem A - RLC circuit analysis

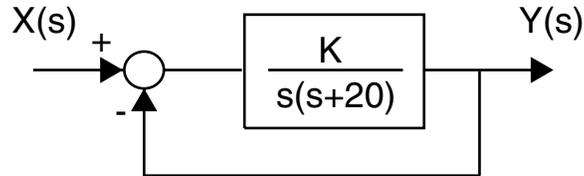


1. Write the transfer function,  $\frac{V_{out}(s)}{V_{in}(s)}$ , for the circuit shown above.
2. Given  $C = 1 \mu F$ , find the values of  $R$  and  $L$  such that  $\xi = 0.707$  and the undamped natural frequency is 5 kHz. (Don't forget to convert to rad/sec!!!)
3. Using the same values of  $L$  and  $C$  from part(2), and the locations of any system pole(s) and zero(s) given  $R = 1000 \Omega$  Sketch the unit step response, clearly indicating the time and magnitude scaling. (Hint: use a dominant pole approximation.) Use the IVT and FVT to show that your response starts and ends at the appropriate values.
4. Sketch the log Magnitude vs log frequency, and linear phase vs log frequency (Bode plot) for this system based on your calculated poles from part 3.
5. Again using  $C = 1 \times 10^{-6} F$  and  $R = 1000 \Omega$  now let  $L = 0 H$ . (i.e. Remove the inductor from the circuit) Calculate the location of the pole and compare this to the dominant pole found in part 2.
6. Write the transfer function,  $\frac{V_{out}(s)}{V_{in}(s)}$ , for the two circuits shown below.

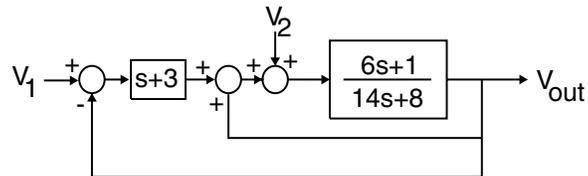


Problem B - Block Diagram

1. Reduce the block diagram below to derive the transfer function for the system. Find the value of  $K$  that will result in a critically damped response.

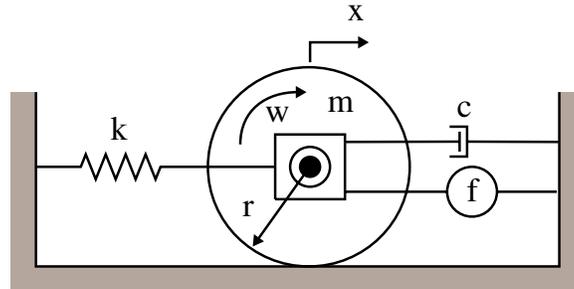


2. Derive the transfer functions  $\frac{V_{out}(s)}{V_1(s)}$  and  $\frac{V_{out}(s)}{V_2(s)}$  for the block diagram shown below. (Hint, you can always label individual positions along the block diagram with variable names and write out and solve the relevant algebraic equations.)

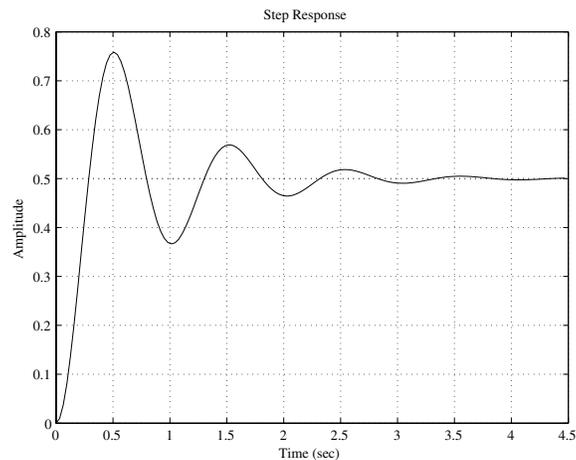


3. Write the complete differential equation for  $V_{out}(t)$  in terms of  $V_1(t)$  and  $V_2(t)$ .

Problem C - Step response

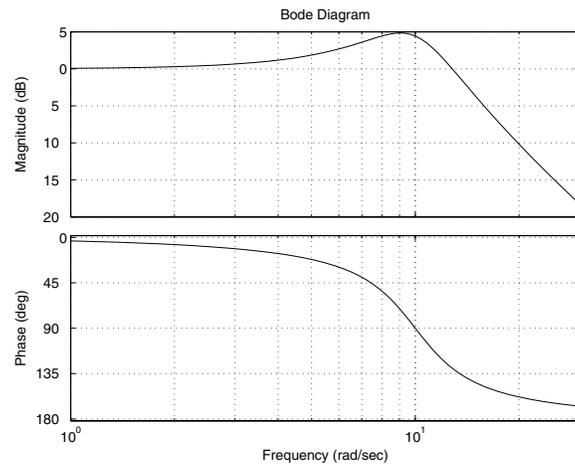
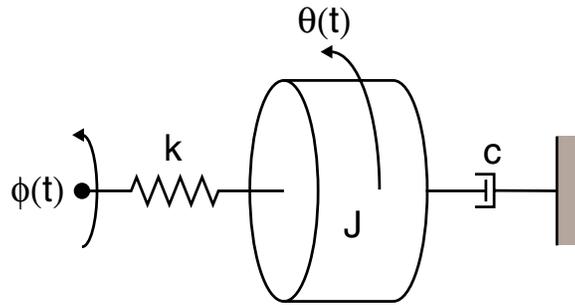


You have been given the system illustrated above. The system consist of a cylinder with a mass ( $m$ ) with a radius ( $r=0.5$  m) which spins about an axle. The cylinder rolls without slip on the ground. Attached to the axle housing are a damper ( $c$ ), a spring ( $k=200$  N/m), and a force source ( $f$ ). You measure the following response  $x(t)$  to a step input of the force source.



- Using the provide parameters and the step response, determine the damping constant ( $c$ ) and the equivalent mass ( $m_{eq}$ ), where  $m_{eq}$  is mass equivalent of the combined inertia and mass.
- If the cylinder has a  $m=3$  kg, determine the inertia of the cylinder.

Problem D



You perform a frequency analysis of the system shown above and obtain the bode plot shown above. The inertia  $J=15 \text{ N/m}^2$ .

1. Using the data in the bode plot determine damping ratio and natural frequency for this system.
2. Using  $J$  and the values determined in part 1, determine  $c$  and  $k$  for the system.
3. Determine an expression for the system output for an input  $\sin(\omega t)$  where  $\omega=1.1, 10,$  and  $20 \text{ rad/s}$ .