

2.003 Problem Set 4

Assigned: Fri. Feb. 25, 2005

Due: Fri. Mar. 4, 2005, in recitation

Note: While this problem set is due on Friday March 4, the first 5 problems are taken from earlier quizzes; working all these problems in advance of Quiz 1 is highly recommended. Each of the first 5 problems is intended to be something you can complete in about 30 minutes in a quiz. We are not providing solutions to these problems. If you cannot work these to your satisfaction, you should definitely consult with course staff or other students to gain mastery of the required material. Problem 6 is somewhat longer, but provides a good practice problem for fluid systems. The last problem is looking for qualitative answers to get you to think physically about the problems you've been solving.

Problem 1 Archive Problem 17.2

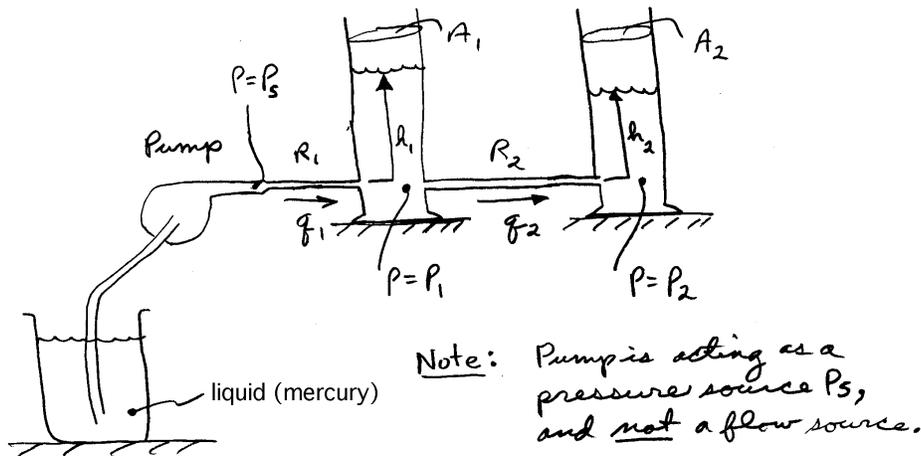
Problem 2 Archive Problem 17.3

Problem 3 Archive Problem 17.15

Problem 4 Archive Problem 17.17

Problem 5 Archive Problem 17.22

Problem 6 This problem considers the fluid system shown below.



As shown in the figure, the system consists of a pump which acts as a *pressure* source. Be sure you realize that this means the pressure at the pump outlet P_s is set by the pump independent of the pump output flow. The pump is connected to the first tank by fluid resistance R_1 ; note that this means that the pressure at the left-hand side of R_1 is equal to P_s . The first tank has cross-sectional area A_1 , and the height of the fluid above the inlet level is defined as h_1 . The first tank is connected to the second tank by fluid resistance R_2 , and the height of the fluid above the inlet level is defined as h_2 . The fluid flows in resistances R_1 and R_2 are q_1 and q_2 , respectively.

All quantities in this problem are in MKS units. All pressures are measured with respect to vacuum. The fluid being pumped is mercury (don't try this at home, kiddies!) with a density of $\rho \approx 13.5 \times 10^3 \text{ kg/m}^3$. The system is also assumed to be subjected to an ambient pressure $P_{atm} = 10^5 \text{ Pa}$.

The numerical values of the parameters are $R_1 = 1.32 \times 10^9 \text{ Pa sec/m}^3$, $R_2 = 1.32 \times 10^{10} \text{ Pa sec/m}^3$, and $A_1 = A_2 = 5 \times 10^{-4} \text{ m}^2$. Recall that the fluidic capacitance of tanks such as in this problem is given by $A/\rho g$ where g is the acceleration of gravity.

- a) Develop a differential equation which describes this system in terms of the pressure $P_s(t)$ as an input, and the height of fluid in the right-hand tank $h_2(t)$ as an output. Plot the system poles in the s -plane. Indicate which of the two poles will have the slower response.
- b) Now assume that the system is initially at rest ($h_1 = h_2 = 0, P_s = P_{atm}$). Further assume that at $t = 0$ the pump pressure takes a step change given by $P_s(t) = P_{atm} + (10^5 u_s(t)) \text{ Pa}$. Write a solution for the response $h_2(t)$, and sketch this response as a function of time.

Problem 7 This question follows the water in the basement problem (6.3) that was due on Friday. This question is intended to explore your qualitative understanding of the course material. We do not expect any detailed analytical developments for this question; rather try to use your physical intuition for what is happening, and answer the question in words rather than equations. Prior to the roller blades breaking the pipe (in engineering we often call this time $t < 0$), water was in the pipe heading from the pressure tank to the shower head. Imagine a very small volume of this water, about the size of a pea (call it v), which at the time of the break ($t = 0$) was at a height equal to the height of the floor of the home on which the shower stall rests, or just a bit below the toes of the person in the shower.

- a) Describe, in no more than a few sentences, the height, with respect to this floor (floor height = 0), of our small volume of water, v , during the one half second prior to the breaking of the pipe.
- b) Sketch a plot of the height of our water volume, v , during this time interval one half second prior to the breaking of the pipe.
- c) Describe, in no more than a paragraph, the height with respect to the floor (floor height = 0) of our small volume of water, v , during the five seconds following the breaking of the pipe.
- d) Sketch a plot that describes the height of the water volume, v , during this 5 second time interval.