

Massachusetts Institute of Technology
 Department of Mechanical Engineering
 2.003 Modeling Dynamics and Control I
 Spring 2005, Prelab 4

Assigned 3/10/05, due at start of lab during week 3/14/05

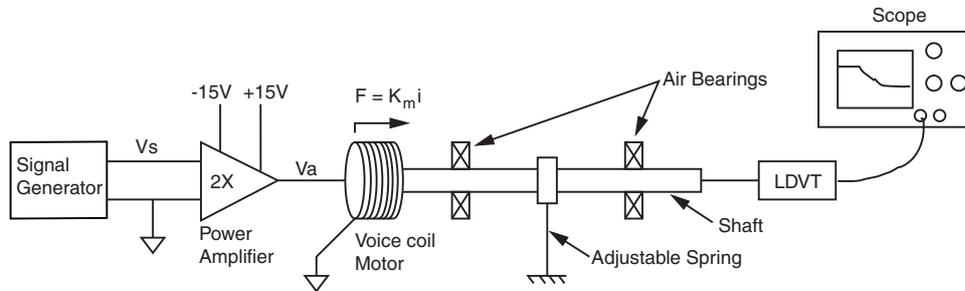


Figure 1: Schematic of Lab setup

For Lab 4, you will be examining the step response of the spring/mass/damper system studied in Lab 3. For this lab, we need to generate a new system model since we are using the voice coil motor to excite the system.

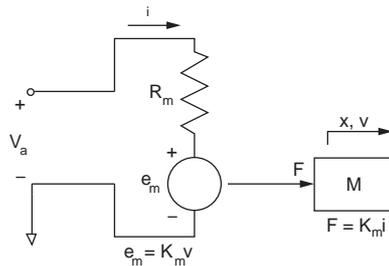


Figure 2: Circuit of voice coil motor driven by the amplifier voltage (V_a)

In general, dc motors (a voice coil motor is a single phase dc motor that produces linear motion) produce a force that is proportional to the current driven through the motor

$$F_m = K_m i \tag{1}$$

where K_m is the motor force constant with units of N/A for linear systems. A dc motor can be driven either by a current or voltage source. For this lab you will be driving the system with a voltage source (the power amplifier). The circuit diagram for this system is shown in Figure 2. In this circuit, V_a is the voltage at the output of the power amplifier, R_m is the electrical resistance of the motor coils, and x is motor displacement (For our purposes, we can

ignore the coil inductance and capacitance). // The actuator introduces a new variable e_m called the back electromotive force (back emf), which is the e_m the voltage generated in the motor coils when the coils are moved through a magnetic field. It can be shown that

$$e_m = K_m \dot{x} \quad (2)$$

$$\text{Units: } [V] \equiv \frac{[J]}{[C]} = \frac{[J/s]}{[C/s]} = \frac{[W]}{[A]} = \frac{[N \cdot m]}{[A \cdot s]}$$

Summing the voltages around the circuit of Figure 2 yields

$$-V_a + i_m R_m + e_m = 0, \text{ or} \quad (3)$$

$$V_a = i R_m + e_m = i R_m + K_m \dot{x} \quad (4)$$

Solving equation (4) for i yields

$$i = \frac{V_a - K_m \dot{x}}{R_m} \quad (5)$$

Substituting i into (1) allows us to determine the voice coil force F_m which acts on the mass as

$$F_m = K_m i = K_m \frac{V_a - K_m \dot{x}}{R} = \frac{K_m}{R} V_a - \frac{K_m^2}{R} \dot{x} \quad (6)$$

The force from the motor contains both a term proportional to the system input V_a and the negative of a term proportional to the velocity of the coil/shaft system. The K_m^2/R term can be interpreted as an equivalent damping constant b_{eq} with units of Ns/m .

As you saw in Lab 3, the back emf is not the only source of damping in the voice coil motor. Another source of damping is the currents flowing in the aluminum cup supporting the motor coils. Since aluminium is conductive, the same physics (Faraday's law of induction) that apply to moving the copper motor coils through a magnetic field apply to the aluminium cup. Moving the cup through the motor's magnetic field causes a voltage potential in the cup. Since the cup is a continuous part, the voltage induces a current (typically termed eddy current) to flow around the cup. The induced current then interacts with the magnetic field to create a force on the cup (Ampere's law). It is possible to analytically calculate a damping constant associated with the eddy currents in the cup but for present purposes we will use the damping constant measured in lab ($b_{eddy} \approx 5 \text{ Ns/m}$).

Including the eddy current damping in our motor model yields

$$F_m = \frac{K_m}{R} V_a - \left(\frac{K_m^2}{R} + b_{eddy} \right) \dot{x} \quad (7)$$

Note that because the amplifier acts as *voltage* source, the damping in the system is the same as with the actuator coil shorted. The amplifier is configured with a gain of 2, which means that $V_a = 2V_s$.

Since non-idealities can distort the initial movement of a system, one commonly used metric to avoid these non-idealities is the 10-90% rise time. For underdamped systems the rise time is approximated by

$$t_r(10 - 90\%) \approx \frac{0.8 + 2.5\zeta}{\omega_n} \text{ for } 0 \leq \zeta \leq 1. \quad (8)$$

Problems

1. Given the motor force characteristics (equation 7), derive the differential equation relating the voltage input from the signal generator v_s to the shaft position x for this system. Be sure to include the power amplifier voltage gain of 2V/V. Write your expression in terms of the system variables.
2. It can be difficult to measure the 2% settling time of a system in the lab. A more commonly used metric is the 5% settling time. Determine the relationship between the system parameters and the 5% settling time (ie. $t_s(5\%) = a\tau$ where a is a constant).
3. Given that

$$\begin{aligned} K_m &= 7.1 \frac{[N]}{[A]} \\ R &= 5.5 [\Omega] \\ m &= 0.85 [kg] \\ k &= \frac{0.132 [N]}{l^3 [m]} \end{aligned}$$

Analytically calculate the step response for spring lengths of 50, 100, and 150 mm. That is, derive an expression for the response $x(t)$ to a 1*Volt* step in V_s , for the three spring lengths. (ie. determine $x(t)$ for each spring length). Use MATLAB to plot each of these time responses.

4. For each response in 3, graphically determine $t_r(10 - 90\%)$, t_p , M_p , and $t_s(5\%)$.
5. Calculate $t_r(10 - 90\%)$, t_p , M_p , and $t_s(5\%)$ directly from the differential equation using the formulas presented in the supplemental notes on 1st and 2nd order systems. Compare these to your graphically determined values.