

2.003 Engineering Dynamics

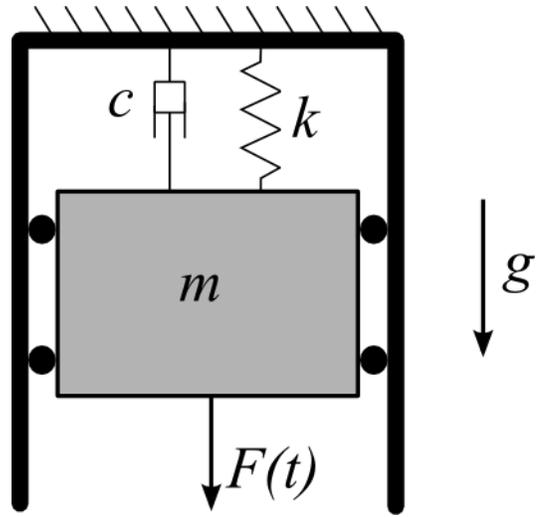
Problem Set 9--Solution

Problem 1

Find the equation of motion for the system shown with respect to:

- Zero spring force position. Draw the appropriate free body diagram.
- Static equilibrium position. Draw the appropriate free body diagram.
- Concept question:* Does g enter into the expression for the undamped natural frequency?

Answer. The acceleration of gravity will not enter into the expression for the natural frequency, because the term involving g in the EOM is a constant and not a function of the motion coordinate, $x(t)$.



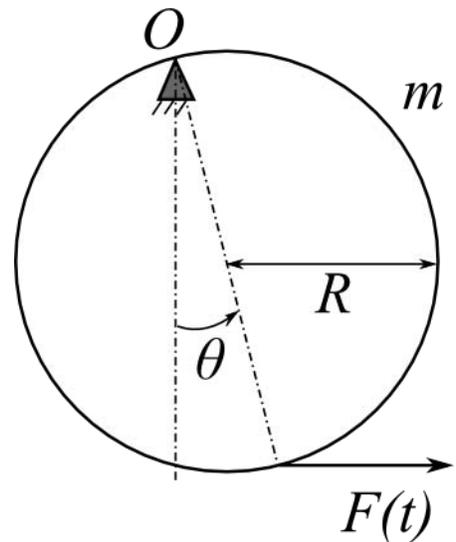
Problem 2

A thin hoop of mass m and radius R is hanging from a knife edge (there is no slip between the hoop and the knife edge). The force $F(t)$ is always horizontal.

- Draw a free body diagram and derive the equation of motion of the hoop.
- Find the linearized equation of motion using a small angle approximation for θ (define θ such that the static equilibrium position is $\theta=0$).
- Find the undamped natural frequency of the system.

Concept question: Does g enter into the expression for the undamped natural frequency?

Answer: In this case the acceleration of gravity appears in a term in the EOM which involves the angle of the motion, θ . The natural frequency expression will include g .



Problem 3

Try to come up with a general rule that will predict when the acceleration of gravity will appear in the expression for the natural frequency. Perhaps compare the systems of problems 1 and 2 to illustrate your answer.

Problem 4

For the system in problem 1, let $K=10,000$ N/m, $M=0.633$ kg, $x_0=0.1$ m, $v_0=10$ m/s and the damping ratio $\zeta =0.05$.

- Find an expression for $x(t)$ in terms of the initial conditions. Express $x(t)$ in the form $x(t) = A \cos(\omega t - \phi)$.
- Sketch $x(t)$ versus time.
- Compute the ratio of the damped to the undamped natural frequency for damping ratios of (i) 5%, (ii) 10% and (iii) 20% of the critical damping.

Concept question: If in an experiment this system was given an initial velocity and observed to decay in amplitude of vibration by 50% in 10 cycles of vibration, can you estimate the damping ratio of the system. i. 0.005, ii. .01, iii,.02?

Answer: The simple rule of thumb says that if $n_{50\%}$ is the number of cycles of motion required to decay in amplitude by 50%, then the damping ratio is given by $\zeta = \frac{0.11}{n_{50\%}} = \frac{0.11}{10} = 0.011$.

Problem 5

For the values of M, K used in the previous problem, compute the steady state amplitude and phase angle of the response to a harmonic force specified as $F(t) = F_o \cos(\omega t)$, where $F_o=10$ N and the damping ratio is 5% of critical. Do this computation for three values of $\omega/\omega_n =$ (i) 0.5, (ii) 1.0 and (iii) 3.0.

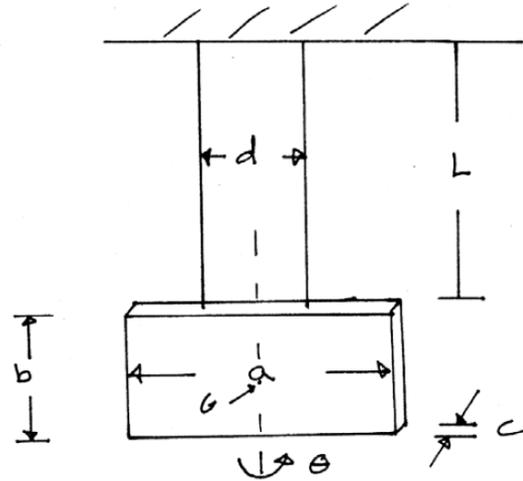
- Concept question:* At which of the three excitation frequency ratios will the response magnitude be greatest. i. $\frac{\omega}{\omega_n} = 0.05$, ii. 1.0, iii 3.0?

Answer: The response will be greatest at resonance, when $\frac{\omega}{\omega_n} = 1.0$.

Problem 6

A block of wood is suspended by two strings, as shown in the Figure below. The strings are separated by 10.0 cm. and are 28 cm in length. $a = 14.0$ cm, $b = 6.0$ cm, and $c = 1.8$ cm.

- Determine the mass moment of inertia with respect to the vertical axis of rotation which passes through the center of mass of the block.
- Find an equation of motion for small torsional oscillations about the vertical axis which passes through the center of mass. Use the direct method in which you sum the external moments to find the equation of motion. Linearize the equation of motion for small oscillations.
- Find expressions for T and V , the kinetic and potential energies of the block in terms of the rotational velocity and the angle of rotation of the block.



Concept question: Can all of the kinetic energy

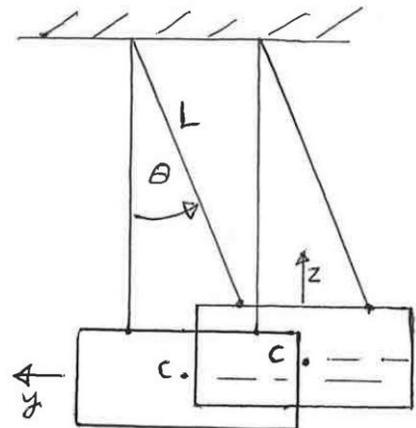
be accounted for by an expression of the form $T = \frac{1}{2} I_{zz} \omega^2$?

Answer: No, there will be some vertical translational velocity of the center of mass and this body is not rotating. I_{zz} will not enter into the solution.

Problem 7

Consider the same system as in problem 6.

- Find the kinetic and potential energy expressions for the block when it swings in the plane of the paper as shown in the figure. This means there is no torsional motion about the vertical axis passing through the center of mass.
- Use the concept of conservation of total energy to find the equation of motion. Linearize the EOM and find the natural frequency.
- Assume that we conduct an experiment with this system. It is given an initial angular deflection of 0.2 radians and released. Over time the oscillation amplitude becomes smaller due to



damping. After 5 cycles of vibration the maximum angle of motion has reduced to 0.08 radians. What is the approximate damping ratio of the system.

d) *Concept question: Will the expression for T have a term of the form $T = \frac{1}{2} I_{zz} \omega^2$?*

Answer: This body has translational velocity only. There is no rotation and therefore no kinetic energy of the form shown in the question above.

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