Problem 1 - Solution

a. Moment of Inertia

The moments of inertia of the rod and the disk about their centers of mass are, respectively:

$$I_G^{rod} = \frac{1}{12} m_1 l^2 \qquad\qquad I_G^{disk} = \frac{1}{2} m_2 r^2$$

From the parallel axis theorem, the moment of inertia of the entire pendulum about point O is:

$$I_O = [m_1(\frac{l}{2})^2 + \frac{1}{12} m_1 l^2] + [m_2(\frac{l}{2})^2 + \frac{1}{2} m_2 r^2]$$

or

$$I_O = \frac{1}{3} m_1 l^2 + \frac{3}{4} m_2 r^2 \tag{1}$$

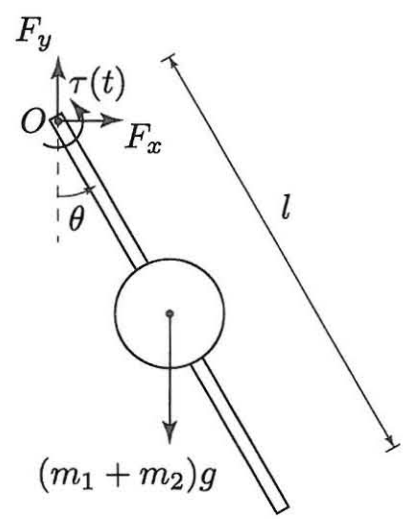
b. Angular Momentum

The angular momentum about point O for the entire pendulum is:

$$H_O = I_O \omega = [\frac{1}{3} m_1 l^2 + \frac{3}{4} m_2 r^2]\dot{\theta} \tag{2}$$

c. Free Body Diagram

The free body diagram for the system contains the weight, the reaction forces and the torque:



Problem 1 solution, continued.

d. Equation of Motion

Because the pivot is fixed, we can sum torques about point O and use the following:

$$\tau_O = \frac{dH_O}{dt}$$

$$\tau(t) - mg\frac{l}{2}sin\theta = I_O\ddot{\theta} \tag{3}$$

or

$$[\frac{1}{3}m_1l^2 + \frac{3}{4}m_2r^2]\ddot{\theta} + mg\frac{l}{2}sin\theta = \tau(t) \tag{4}$$

Quiz2 Problem 2 solution

solution: Need $\underline{\omega}$ in body coordinate

$$\omega = \omega_x \hat{i} + 0\hat{j} + \omega_z \hat{k}$$

a) $${}^{G}H = {}^{G}\underline{\underline{I}}\,\underline{\omega} = I_{xx}\omega_x i + 0\cdot j + I_{zz}\omega_z k$$

where $$\Omega = \frac{\sqrt{2}}{2}\Omega\hat{i} + \frac{\sqrt{2}}{2}\Omega\hat{k}$$

$$= \omega_x \hat{i} + \omega_z \hat{k}$$ expressed in components in the rotating frame G_{xyz}.

b) $${}^{G}\underline{\tau}_{ext} = \frac{d\,{}^{G}H}{dt} = \left(\frac{\partial\,{}^{G}H}{\partial t}\right)^{G_{xyz}} + {}^{o}\omega \times {}^{G}H$$

$$= I_{xx}\mathring{\omega}_x i + \cancel{I_{xx}\omega_x i} + I_{zz}\mathring{\omega}_z k$$

$$+ (\omega_x i + \omega_z k) \times (I_{xx}\omega_x i + I_{zz}\omega_z k)$$

$$\tau_{ext} = I_{xx}\mathring{\omega}_x i + I_{zz}\mathring{\omega}_z k$$

$$- I_{zz}\omega_x\omega_z j + I_{xx}\omega_x\omega_z j$$

c + d It is statically balanced but not dynamically balanced, Problem 3, Solution

a. False

b. True

c. True

Problem 4

$$\boxed{\text{a)}\quad T = \underbrace{\tfrac{1}{2} I_{zz}\, \omega_z^2}_{\text{rod}} + \underbrace{\tfrac{1}{2} I_{zz}\, \omega_z^2}_{\text{tube}} + \tfrac{1}{2} m\, \underline{v}_p \cdot \underline{v}_p \qquad v_p = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$

$$v_p \cdot v_p = \dot{r}^2 + r^2\dot{\theta}^2$$

$$\boxed{\text{b}\quad V = 2\cdot\tfrac{1}{2}k r^2 + \tfrac{1}{2}k_T\theta^2}$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\theta}} = I_{zz_r}\ddot{\theta} + I_{zz_T}\ddot{\theta} + \frac{d}{dt}\left(m r^2\dot{\theta}\right)$$

1 $$= \left(I_{zz_r} + I_{zz_T}\right)\ddot{\theta} + m r^2\ddot{\theta} + 2 m r\dot{r}\dot{\theta}$$

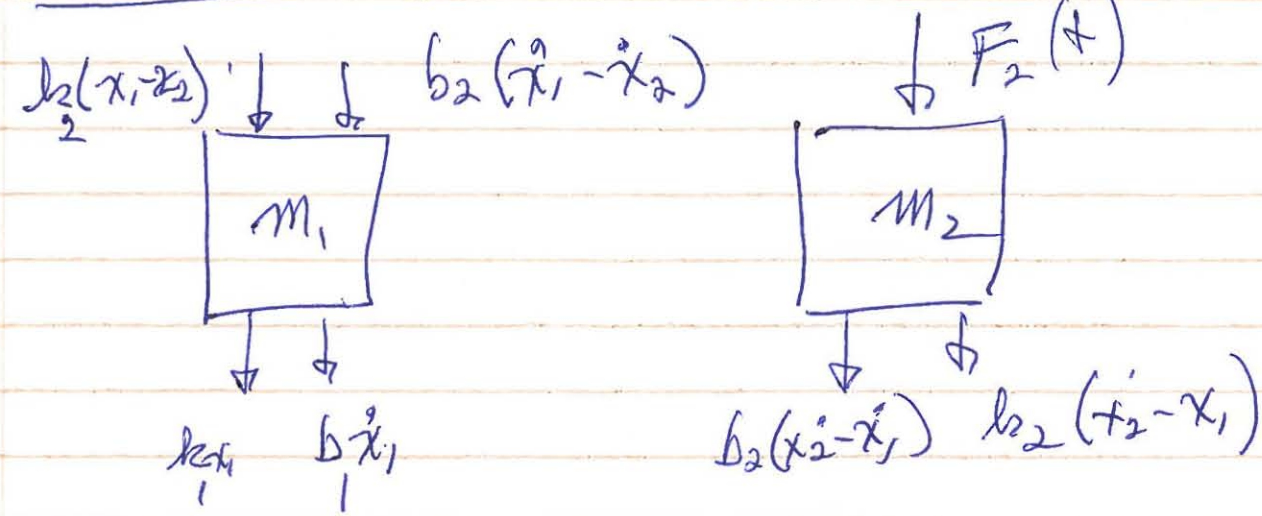
2 $-\dfrac{\partial T}{\partial\theta} = 0$ 3 $+\dfrac{\partial V}{\partial\theta} = k_T\theta$ 4 $Q_\theta = 0$

2) $$\boxed{\left(I_{zz_r} + I_{zz_T} + m r^2\right)\ddot{\theta} + k_T\theta = 0}$$

$\dfrac{d}{dt}\dfrac{\partial T}{\partial\dot{r}} = m\ddot{r}$ $-\dfrac{\partial T}{\partial r} = -2r\dot{\theta}^2 m$ $+\dfrac{\partial V}{\partial r} = 2kr$

$$\boxed{m\ddot{r} - 2r\dot{\theta}^2 m + 2kr = 0}$$

Problem 5 fbd's



$$Q_{x_1} = -b_1\dot{x}_1 - b_2(\dot{x}_1 - \dot{x}_2)$$

$$Q_{x_2} = -b_2(\dot{x}_2 - \dot{x}_1) - F(t)$$