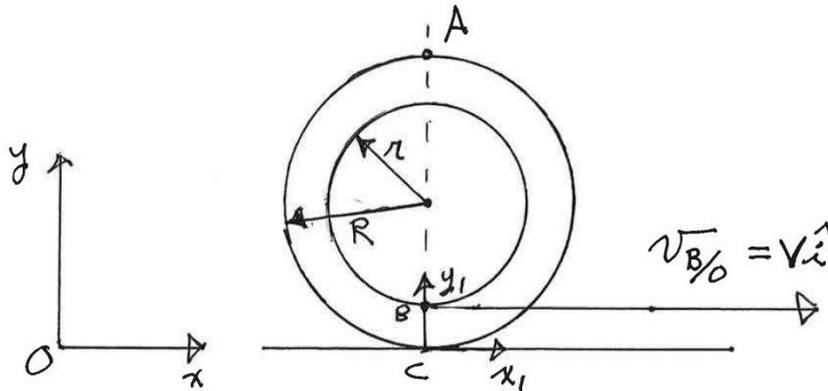


**2.003 Engineering Dynamics**  
**Problem Set 4**

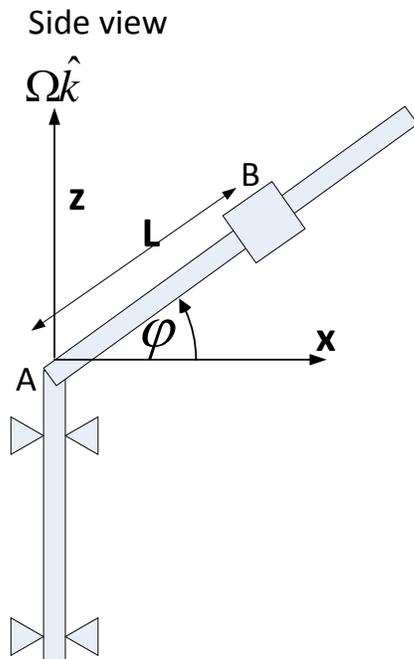
**Problem 1:**

1. Determine the velocity of point A on the outer rim of the spool at the instant shown when the cable is pulled to the right with a velocity of  $v$ . Assume  $r < R$  and that the spool rolls without slipping.



**Concept question:** Will the spool roll to the left or to the right when the string is pulled to the right? a) Left, b) Right, c) Not enough information given.

**Problem 2:**

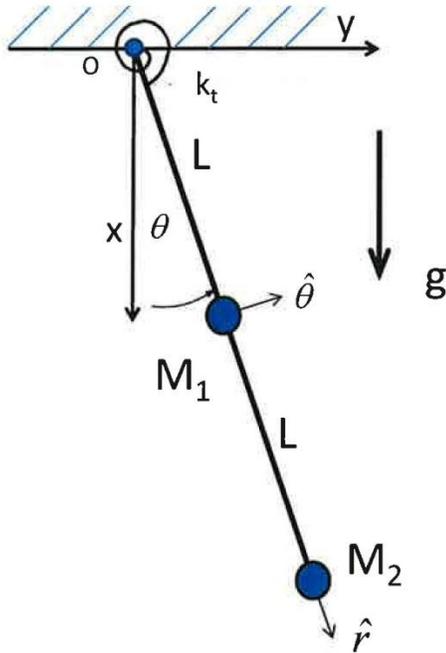


The 2-kg spool S fits loosely at B on the rotating inclined rod for which the coefficient of static friction is  $\mu_s = 0.2$ . If the spool is located a distance  $L$  from A, where  $L = 0.25\text{m}$ , determine the maximum constant rotation rate  $\Omega$ , about a vertical axis passing through A, such that the spool does not slip up the rod. Let  $\phi = 30^\circ$ .

**Concept question:** If the starting position of the spool is moved down the rod to 0.15 m, will the angular rate be higher when the spool begins to slide up the rod? (a) Yes, (b) No, (c) Not sure.

**Problem 3.**

This is a simple pendulum with a torsional spring at the pivot. Gravity acts. The torsional spring creates a restoring torque, which is proportional to the angle of rotation of the rod. The rod is of length  $2L$ , rigid and without mass. Two masses are attached to the rod, one at the midpoint and one at the end. For the purpose of this problem the masses may be considered to be particles with mass  $M$ .



(a) The masses and the rigid rod make up one rigid body, which has at most 6 degrees of freedom. However, in this problem the rigid body has 5 external constraints. Thus it has only one degree of freedom and will require only a single coordinate to completely describe its motion. Identify the 5 constraints.

(b) Find the equation of motion of this pendulum by consideration of the time rate of change of the angular momentum computed with respect to the pivot. Be sure to include a free body diagram.

Concept Question:

The natural frequency of the pendulum without a torsional spring is independent of mass and equal to

$$\omega_n = \sqrt{\frac{2g}{3L}}$$

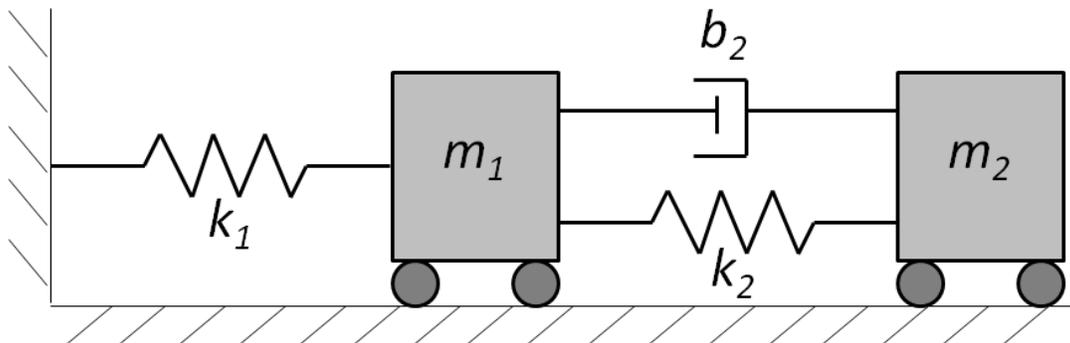
How will the natural frequency change as a result of the addition of the torsional spring. (a) Increase, (b) Decrease, (c) Stay the same.

#### Problem 4:

This problem has multiple bodies. It consists of two masses, with springs and a dashpot, mounted on rollers on a horizontal surface. Unlike particles which are assumed to be points of mass, rigid bodies have finite dimensions and therefore rotational inertia. Each rigid body has six possible degrees of freedom: three in translation and three in rotation. The number of degrees of freedom is given by the expression  $DOF = 6N - C$ , where  $N$  is the number of rigid bodies and  $C$  is the number of constraints.  $DOF$  is the number of independent degrees of freedom. For each  $DOF$  you need to assign a coordinate to describe the motion. For most mechanical systems the number of  $DOFs$  equals the number independent coordinates needed to completely describe the motion of the system. When this is true the systems are called holonomic. More on that topic will be dealt with later in the course. This system is holonomic. You will use the equation above to establish the number of independent coordinates needed to solve this problem.

(a) In this problem there are 2 rigid bodies and a possible 12 degrees of freedom. However, there are 10 constraints, which leads one to the conclusion that only two independent coordinates are needed to describe the motion. Describe the ten constraints that reduce the number of degrees of freedom to two.

(b) Draw free body diagrams, assign two appropriate coordinates, and find the two equations of motion which characterize the system. Hint: To get the signs correct on the spring and dashpot forces, one at a time assume positive displacements and velocities of each coordinate and deduce the resulting directions of the spring and damper forces. Draw these forces on your free body diagrams. It is also strongly suggested that you assign coordinates so that they will be zero when the system is in its static equilibrium position.



**Concept question:** If spring constant  $k_1$  were zero, the object is no longer constrained in the horizontal direction. The system still has two natural frequencies.

Do you think it will be possible for the two masses to vibrate(oscillate) in such a way that the center of mass of the system does not move. (a) Yes (b) No (c) Not sure

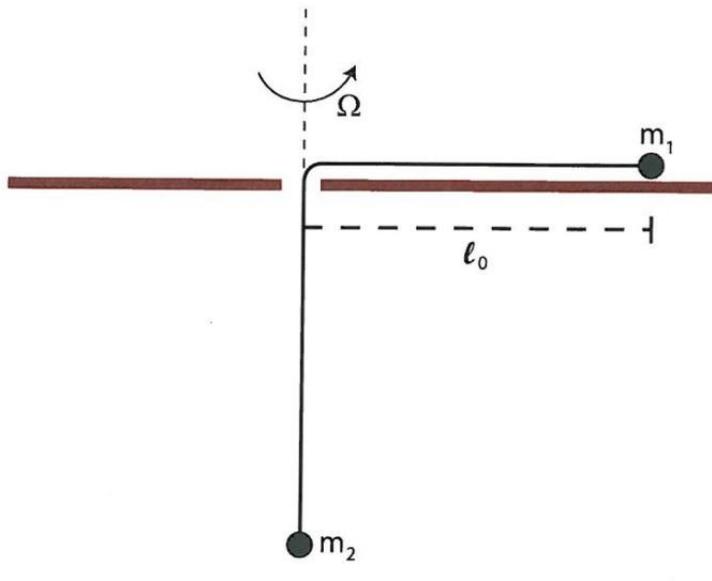
**Problem 5.**

A mass  $m_1$ , attached to a string, is on top of a frictionless table. The mass is rotating about a hole in the table through which the string passes. The initial rate of rotation is  $\Omega$  and the initial distance from mass  $m_1$  to the hole is  $l_0$ . At the other end of the string, lying exactly on the axis of rotation, is another mass,  $m_2$ . Note that gravity acts and that the contact of the string with the hole is without friction.

- a) Determine the number of degrees of freedom for this problem.
- b) Find the equation(s) of motion.
- c) Based on initial conditions given will the mass  $m_1$  begin moving inward or outward?

**Concept question:** Is total system energy conserved in this problem after it is released?

- (a) Yes, (b) No, (c) Depends on initial conditions



MIT OpenCourseWare  
<http://ocw.mit.edu>

2.003SC / 1.053J Engineering Dynamics  
Fall 2011

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.