

## Problem Set No. 6

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**Problem 1**

**Rod sliding down wall.** The two ends of the rigid rod of mass  $M$  and length  $L$  are in contact with the floor and a vertical wall. Assuming that friction is negligible, derive the equation(s) governing the sliding down of the rod.

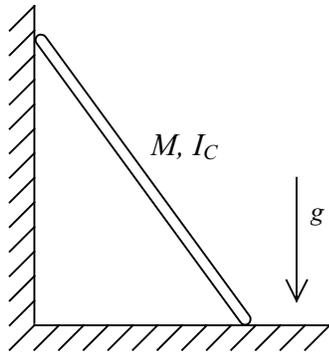


Figure 1: Rod slides down wall

### Problem 2

**Unbalanced disk rolls down incline.** A particle of mass  $m$  is embedded at a distance  $l$  from the center  $O$  of a rigid massless disk of radius  $r$ , which rolls without slipping down a plane inclined at an angle  $\alpha$  with the horizontal. Derive the differential equation governing the generalised coordinate  $\theta$  shown below.

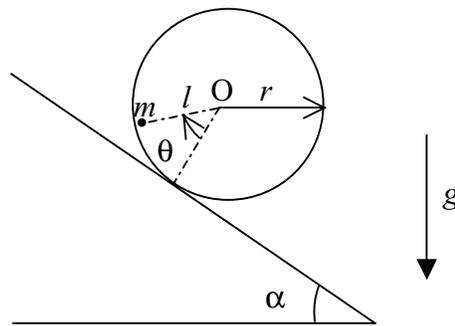


Figure 2: Mass particle  $m$  embedded in massless disk

### Problem 3

**Mechanical model of tight-rope walker.** Professional tight-rope walkers have learned how to use visual signals, and signals from the neurons in the semi-circular canals of the inner ear (which are sensitive to angular motion) to stabilize their precarious position by exerting torques on the long balancing poles which they carry. These poles are usually hollow tubes of bamboo with the end sections filled with lead to increase their mass and moment of inertia. To obtain some insight into the sophistication of the control process that the tight-rope walker has taught herself, or himself, we propose to study a highly simplified mechanical model of the walker and her, or his, balancing pole. In this problem the goal is to derive the equations of motion of the unstable model. In a later problem set we will return to this model and design a control system which is capable of stabilizing the model.

In Fig.4 we consider a simplified mechanical model which behaves somewhat like a skilled

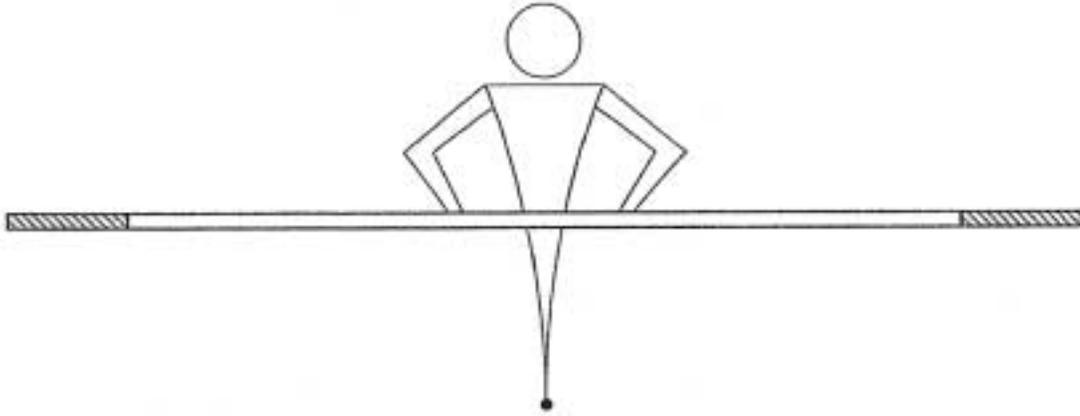


Figure 3: Tight-rope walker and balancing pole.

tight-rope walker. The tight rope is considered to be a fixed, frictionless, pivot  $O$  about which the performer, modeled as a rigid body with mass  $m_1$  and moment of inertia  $I_1$  about its center of mass  $C$ , can balance by exerting torque  $\tau$  on the pole which has mass  $m_2$  and moment of inertia  $I_2$  about its center of mass.

For simplicity, it is assumed that the pole is held so that the two centers of mass coincide in the plane of the sketch. The distance between the pivot  $O$  and the mass center  $C$  is  $a$ . The control system which is subsequently to be designed will apply a torque  $\tau(t)$  between the two rigid bodies based on sensor signals that indicate the values of the angles  $\theta(t)$  and  $\phi(t)$ . At this stage we simply take  $\tau(t)$  to be a prescribed function of time. When  $\tau(t)$  is positive, the torque on the rod with moment of inertia  $I_2$  is counter-clockwise and its equal and opposite reaction on the rigid body with moment of inertia  $I_1$  is clockwise.

A skilled tight-rope walker is able to maintain very small fluctuations in the angles  $\theta$  and  $\phi$  throughout the performance of her, or his, act. It therefore is appropriate to carry out the analysis under the small angle assumptions

$$\sin \theta \approx \theta \quad \text{and} \quad \cos \theta \approx 1$$

- (a) Using linearized expressions, correct to first order in the small angles  $\theta$  and  $\phi$ , obtain the displacement and velocity components of the point  $C$ .
- (b) Draw clear free-body diagrams of the rigid body representing the performer, and the

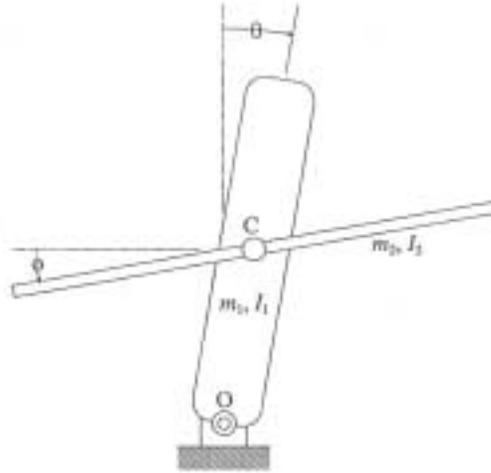


Figure 4: Mechanical model of tight-rope walker and balancing rod.

rigid body representing the balance pole, showing all forces acting on each body.

- (c) Use the linear momentum equations to evaluate each reaction-force component in terms of operations on the angles.
- (d) Apply angular momentum principles to obtain equations of motion for  $\theta(t)$  and  $\phi(t)$ .