## Summary: Root Locus sketching rules

## **Negative Feedback**

- **Rule 1:** # branches = # poles
- Rule 2: symmetrical about the real axis
- **Rule 3:** real-axis segments are to the left of an *odd* number of real-axis finite poles/zeros
- Rule 4: RL begins at poles, ends at zeros
- **Rule 5:** Asymptotes: real-axis intercept  $\sigma_a$ , angles  $\theta_a$

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}} \qquad \theta_a = \frac{(2m+1)\pi}{\#\text{finite poles} - \#\text{finite zeros}} \qquad m = 0, \pm 1, \pm 2, \dots$$

• **Rule 6:** Real-axis break-in and breakaway points

Found by setting  $K(\sigma) = -\frac{1}{G(\sigma)H(\sigma)}$  ( $\sigma$  real) and solving  $\frac{\mathrm{d}K(\sigma)}{\mathrm{d}\sigma} = 0$  for real  $\sigma$ .

• **Rule 7:** Imaginary axis crossings (transition to instability)

Found by setting 
$$KG(j\omega)H(j\omega) = -1$$
 and solving 
$$\begin{cases} \operatorname{Re}\left[KG(j\omega)H(j\omega)\right] &= -1, \\ \operatorname{Im}\left[KG(j\omega)H(j\omega)\right] &= 0. \end{cases}$$

## **Sketch the Root Locus**

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Please see: Problem 8.2 in Nise, Norman S. Control Systems Engineering. 4th ed. Hoboken, NJ: John Wiley, 2004.



## Are these Root Loci valid? If not, correct them

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