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2.007 Design and Manufacturing I Spring 2009

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#### 2.007 – Design and Manufacturing I Gears: Terminology, Geometry, Gear Trains, Strength



#### Presented by Dan Frey on 17 MAR 2009

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### Today's Agenda

- Distribute homework #3
- Gears
  - Applications
  - Types
  - Terminology / nomenclature
  - Congugate action
  - Involute curve
  - Analysis & design

#### **Applications of Gears**



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Sobel, Dava, Longitude

### Spur Gears

- Transmit motion between parallel shafts
- Teeth are parallel to the axis of rotation
- This is the simplest kind of gear we'll consider and most of today is dedicated to them





### Other Types of Gears

#### Helical



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Image courtesy of <u>perlmonger</u> on Flickr.



#### Bevel

Rack





#### Worm

#### Early Gears

Drawings of waterwheels and gears removed due to copyright restrictions.

Roman watermills at Barbegal 300AD

Application for powering textile machinery 18<sup>th</sup> century

### Conjugate Action

NOTE: As discussed in class, the rotation of the driven body B can be inferred. The key is that the two bodies stay in contact even after a small increment of rotation. For this reason, the "gear ratio" is a function of the orientation of the line normal to the point of contact.



Let's say  $\omega_A$  is a constant with time. Can we synthesize a shape of body B so that  $\omega_B$  is also constant with time?

Yes, generally one can synthesize a shape B for any shape A to attain conjugate action. An analog of a "rack cutting" procedure would generally do it.

Line of action, also called 'Pressure line'. The line along which the force between two meshing gear teeth is directed. It has the same direction as the force vector. In general, the line of action changes from moment to moment during the period of engagement of a pair of teeth. For involute gears, however, the tooth-to-tooth force is always directed along the same line -- that is, the line of action is constant. this implies that for involute gears the path of contact is also a straight line, coincident with the line of action -- as is indeed the case.



**Pitch point** (p). The point where the line of action crosses a line joining the two gear axes.

http://en.wikipedia.org/wiki/Gear



circles touch, there is purely rolling.

Draw the velocity vectors at the pitch point.

### **Rack Cutting**



Figure by MIT OpenCourseWare.

- A way to get the relative motion you want
- Pick one shape as you wish
- Enforce the motion you want
- Cut away everything that interferes

### **Involute Profile**

- How it is constructed
  Demo
- Properties
  - Conjugate action
  - Allows design of whole sets of compatible gears
  - Conjugate action not sensitive to center distance variations



### More Gear Terminology

From Shigley and Mischke

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This geometry is not an involute.

#### **Pressure Line**

- Where the teeth contact, the surface normal defines a pressure line
- The force transmitted acts along this line
- The pressure line always includes the point of tangency between the pitch circles
- With the involute gear profile, the pressure line is constant



Figure by MIT OpenCourseWare.

From Shigley and Mischke

### Gear Terminology

"Line of action" & "pressure line" & "generating line" are all synonymous

Source: Fig. 1.1 in "Gears." <u>Design and Application of</u> <u>Small Standardized components Data Book 757</u>. Stock Drive Products, 1983. Accessed September 18, 2009. Courtesy of Stock Drive Products/Sterling Instrument.



#### Pressure Angle

- The pressure line acts at some angle to the tangent of the pitch circles
- This angle can be chosen by the designer
- It affects
  - Separation forces
  - Tooth shape



Figure by MIT OpenCourseWare.

#### From Shigley and Mischke

#### **Concept Question**



Courtesy of W. M. Berg, Inc. Used with permission.

A pair of gears are mated. One is driven at a set torque, the other is regulated at a set speed. The gears are the ones circled. What is the ratio of the separation forces and the total force on the bearing?

- 1. << 0.3
- 2. About 0.3
- 3. About 0.5

#### 4. >> 0.5

Answer = 1: The key thing is pressure angle which is 20 deg. The ratio of separation and total force is sin 20 deg.

#### **Contact Ratio**



contact ratio = length of arc of action / pitch = average number of teeth engaged

#### Interference



Figure by MIT OpenCourseWare.

From Shigley and Mischke

#### Backlash



303 Stainless Steel

Courtesy of W. M. Berg, Inc. Used with permission.

### **Gear Selection**

- Pitch
- Face width
- Material
- Pressure angle
- # of teeth
- Hub style, bore, etc.



Courtesy of W. M. Berg, Inc. Used with permission.



Courtesy of W. M. Berg, Inc. Used with permission.

You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of 0.32 inches. They will probably say:

- 1. OK, no problem
- 2. OK, but it will cost a lot
- 3. No, this is not technically possible

I'd say "3". A 48 pitch gear of 1 inch pitch dia has 48 teeth. The requested gear has 0.32\*48=15.4 teeth. Integer s are better for the number of teeth.



Courtesy of W. M. Berg, Inc. Used with permission.

You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of half the smallest one in the catalog. They will probably say:

- 1. OK, no problem
- 2. OK, but it will cost a lot
- 3. OK, but it will be weak
- 4. No, this is not technically possible

I'd say both "2" and "3". A 48 pitch gear is listed with 14 teeth. Half the dia will give a pinion with 7 teeth. It will be hard to make it and tricky to avoid lots of undercut.

### Ways Gears Fail

#### Exceed endurance limit in bending

#### Exceed static yield stress in bending



Image courtesy of <u>deltaMike</u> at Flickr.

#### Exceed endurance limit in contact stress

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#### **Stress in Gears**

Image removed due to copyright restrictions. Please see p. 1 in <a href="http://courses.washington.edu/mengr356/daly/Gear\_stress.pdf">http://courses.washington.edu/mengr356/daly/Gear\_stress.pdf</a>

#### A Beam in Bending



Figure by MIT OpenCourseWare.

#### **Concept Question**

- In selecting a gear of one inch pitch diameter, we are choosing between 48 and 24 pitch gear teeth. The effect on torque that can be transmitted before <u>bending</u> failure of the teeth is
- 1. Around a factor of 10
- 2. Around a factor of 4
- 3. Around a factor of 2
- 4. Less than a factor of 2



Figure by MIT OpenCourseWare.

Answ= 3: Model the gear as a beam. Its depth goes down by a factor of 2 which by itself raises stress by a factor of 4. But the length of the beam also drops by a factor of 2 and that reduces stress linearly.

# Strength of Gears

- Any good catalog will have a formula and tables
- What factors must enter the equation?

• Where do the teeth wear the most?

Gear wear the most where they experience sliding motion the most. That's away from the pitch circle.

#### Gear Reference Guide

#### GEAR TOOTH STRENGTH

Many factors must be considered when designing a gear train. The information listed on this pag should be used as a general guideline for your application. If more critical strength calculation required W.M. Berg suggests that you consult our engineering department or any one of the many gea handbooks that are readily available.

When a gear train is transmitting motion, it is save to assume that all of the load is being carried b one tooth. This is because as the load approaches the end of the tooth, where the bending force would be the greatest, a second tooth comes into mesh to share the load. Simple results can be obtained from the Lewis bending strength equation.

#### $W_t = \frac{SFY}{D.P.}$

- Wt = Maximum transmitted load (lbs or N)
- S = Maximum bending tooth stress (taken as 1/3 of the tensile strength) See Table C on Page 5
- F = Face width of gear (in. or mm)
- D.P. = Diametral Pitch = 1/module (for equation only) Y = Lewis Factor (See Table)



NOTE: The maximum bending tooth stress (S) is valid for well lubricated, low shock applications. For high shock, poorly lubricated applications, the safe stress could be as low as .025S. If your design calls for an unfriendly environment for gears, you might want to lower S to assure a reasonable amount of gear life.

	NO. OF TEETH	14 1/2" INVOLUTE	20° INVOLUTE
LEWIS FACTOR - Y	10	0.176	0.201
	11	0.192	0.226
	12	0.210	0.245
	13	0.223	0.264
	14	0.236	0.276
	15	0.245	0.289
	16	0.255	0.295
	17	0.264	0.302
	18	0.270	0.308
	19	0.277	0.314
	20	0.283	0.320
	22	0.292	0.330
	24	0.302	0.337
	26	0.308	0.344
	28	0.314	0.352
	30	0.318	0.358
	32	0.322	0.364
	34	0.325	0.370
	36	0.329	0.377
	38	0.332	0.383
	40	0.336	0.389
	45	0.340	0.399
	50	0.346	0.408
	55	0.352	0.415
	60 65	0.355	0.421
	70	0.358	0.425
	70	0.360	0.429
	80	0.361	0.433
	90	0.363	0.436
	100	0.366	0.442
	150	0.368	0.446
	200	0.375	0.458
	300	0.378	0.463
	RACK	0.382	0.471

#### 1-800-232-BERG

Berg Manufacturing "The Mark of Quality"

Courtesy of W. M. Berg, Inc. Used with permission.

#### The Lewis Formula







Point of max stress due to bending

#### Or Use a Canned Tool

Please see "Spur Gear Tooth Strength" at http://www.wmberg.com/tools/

http://www.wmberg.com/Tools/

#### **Discussion Questions**

- I glued the third stage teeth of this servo together
- Now I will apply a load to the output shaft (up to 10lbs)
- What's going to happen?



The gears nearest the output shaft will fail since they experience the highest loads. The mode for Delrin gears is most probably in bending or "stripping". I would estimate that the smaller gear (black in the photo) will fail rather than the white one since it has a narrower base and so a lower Lewis factor.

#### **Concept Question**

- For a gear to provide the highest strength at a fixed diameter, we prefer
- 1. High pressure angle
- 2. Low pressure angle
- 3. It doesn't matter much



Figure by MIT OpenCourseWare.

Answ =1. High pressure angle will help raise the Lewis factor and lower stress. Comparing 14.5 deg and 20 deg pressure angle (back a couple slides), the difference is 10 to 20%, so not a negligible difference.

#### Contact Stress (Hertzian Stress)



#### Contact Stress Quantitative Characterization



#### Simple Gear Trains

- A "simple" gear train has only one gear on each shaft
- How does this arrangement behave?

The gears turn in alternating directions. Such an arrangement might be useful to get a motion the same direction as that of the servo but at a different rate. The speed of the driven (yellow) one is not a function of the dia of the middle (pink) one.



### **Compound Gear Trains**

- A "compound" gear train has at least one shaft with multiple gears
- How does this arrangement behave?

The key thing is that the total reduction ratio is a product of the ratios of the two mating pairs.



Image from Wikimedia Commons, http://commons.wikimedia.org

#### Manual Transmissions



Image from Wikimedia Commons, http://commons.wikimedia.org.

Please see http://mossmotors.com/Graphics/Products/Schematics/SPM-025.gif

## If you find just two axles in a machine, does that nean there are just two stages?

No, you probably have a "reverting" arrangement with many compound gears sharing a single pair of shafts.

### **Discussion Questions**



Image from Wikimedia Commons, <u>http://commons.wikimedia.org</u>

- Are there any disadvantage to a helical gear as compared to a spur gear?
- How can the disadvantages be remedied?
- Is a helical gear set stronger than a spur gear of the same diameter, pitch, face width, & material?

Yes, helical gears cause axial loads. They are also more expensive. You can pair up helical gears to cancel the axial loads resulting in a "herring-bone" pattern. Helical gears are not really stronger, they are mostly quieter and smoother.

### **Concept Question**

A compound gear train is formed of eight gears. As we proceed from the pinion on the electric motor to the gear on the output shaft, how do the pitch and face width vary?



- 1. Pitch rises, face width rises
- 2. Pitch rises, face width falls



- 3.) Pitch falls, face width rises
- 4. Pitch falls, face width falls

# Discussion Questions

- How many stages in this device? 4 stages more than 10 to 1
- How do you suppose this number is chosen? ratios per stage are practical.
- Are the reduction ratios typically all nearly the same in all successive stages? No, less reduction ratio as we approach the output shaft.



#### Differentials

- Allows shafts to move at different speeds
- Applies same torque to both
- Slippage problem

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### Next Steps

- Begin Homework #3
- Next lecture Thursday 19 March
  CAD case study
- Spring break
- Lecture Tuesday 31 March
  More gears, and also springs
- HW#3 due 7 April
- Quiz #2 on 16 April
- Impounding week 29 April to 1 May

### **Planetary Gear Trains**

- One or more of the gear axes are allowed to rotate
- aka "epicyclic"
- Used in
  - Power tools
  - Automatic transmissions
  - Gear boxes



Courtesy NASA.

Please also see http://commons.wikimedia.org/wiki/File:Epicyclic\_carrier\_locked.png http://i.i.com.com/cnwk.1d/i/ss/2007/0828\_Driving\_it/DSG\_440.jpg

#### Analysis of Planetary Gear Trains



Figure by MIT OpenCourseWare.

#### Name That Gear



What type of worm gear set is this?

- 1) Single-enveloping, single threaded
- 2) Single-enveloping, multi-threaded
- 3) Double enveloping single threaded worm gear
- 4) Double enveloping multi- threaded

#### Follow up



What is the reduction ratio of this gear set?

- 1) 10:1
- 2) 20:1
- 3) 40:1
- 4) 80:1