Student name: $\qquad$

This is a closed book examination. You are allowed 1 one-sided sheet of 8.5 " x 11 " paper with notes.
For the problems in Section A, fill in the answers where indicated by $\qquad$ or in the provided space. When a list of options is presented ([...],[..],[...] etc), circle all the options (all, none, one or more) that apply.

For part B problems: BE SURE TO SKETCH THE PROBLEM, STATE YOUR ASSUMPTIONS AND SHOW YOUR WORK! Use the provided exam booklet.

Use the following constants unless otherwise specified:

$$
\begin{array}{llll}
\text { Gravity: } \mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2} & \text { water density: } & \rho_{\mathrm{w}}=1000 \mathrm{~kg} / \mathrm{m}^{3} & \text { kinematic viscosity: } v_{\mathrm{w}}=1 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s} \\
& \text { Seawater density: } \rho_{\mathrm{w}}=1025 \mathrm{~kg} / \mathrm{m}^{3} & \text { kinematic viscosity: } v_{\mathrm{sw}}=1 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s} \\
& \text { Air density: } \quad \rho_{\mathrm{a}}=1 \mathrm{~kg} / \mathrm{m}^{3} & \text { kinematic viscosity: } v_{\mathrm{a}}=1 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}
\end{array}
$$

Assume the fluid is incompressible unless otherwise defined.
Give all answers in SI units (kg, m, s). All numerical answers MUST have the proper units attached.

## Part A: (25\%)

1) The boundary condition for the wave equation at the ocean floor is $\qquad$ (give mathematically).
2) An offshore oil platform in the gulfstream capsizes during a hurricane. The operators are unable to cap the oil well and it is leaking oil at a steady rate. The oil is being carried away by the current, the trace of oil represents a [streakline] [pathline] [streamline] [all of these] [none of these].
3) The skipper falls off of a sailing vessel in the southern ocean. His crew does not notice immediately, so he drifts with the current for some time. A satellite overhead is taking a long exposure photograph of the exact area where he was lost and luckily the bright reflector on his lifevest is picked up in the satellite image. The trace made by the reflector represents a [streakline] [pathline] [streamline] [all of these] [none of these].
4) A flow is steady if [ $\partial / \partial t$ ] [D/Dt] [both $\partial / \partial t$ and $\mathrm{D} / \mathrm{Dt}]$ [either $\partial / \partial t$ or $\mathrm{D} / \mathrm{Dt}]$ of the flow variables are zero.
5) The velocity field for a certain flow is given as $V=(u, v, w)$, where $u=4 z+C x^{2}+7$, $v=3 x y+z x$, and $w=4 x^{2}+12 y^{2}+5 x z$. Then constant C must equal $\qquad$ for this to be a valid flow field.
6) A wave with amplitude $\mathrm{a}=1 \mathrm{~m}$ is propagating in a tank. The tank depth is 10 meters, and the wave frequency is 0.35 Hz . These waves are [deep] [shallow] water waves. The appropriate dispersion relationship is given by $\omega^{2}=$ $\qquad$ . The wavelength is $\lambda=$ $\qquad$ , phase speed $\mathrm{V}_{\mathrm{p}}=$ $\qquad$ and group speed $\mathrm{V}_{\mathrm{g}}=$
$\qquad$ .
7) A wave is propagating in a tank with depth 1.12 meters. The wave frequency is 5.6 $\mathrm{rad} / \mathrm{s}$. These waves are [deep] [shallow] water waves. The appropriate dispersion relationship is given by $\omega^{2}=$ $\qquad$ . The wavelength is $\lambda=$ $\qquad$ phase speed $\mathrm{V}_{\mathrm{p}}=$ $\qquad$ and group speed $\mathrm{V}_{\mathrm{g}}=$ $\qquad$ .
8) The pressure on a submarine hatch located at $\mathrm{z}=-100$ meters under a still free surface is $\qquad$ .
9) The dynamic pressure in a steady flow is equal to $p_{d}=$ $\qquad$ , in terms of the velocity, $V$, and the dynamic pressure under linear free surface waves (with no current) is $\mathrm{p}_{\mathrm{d}}=$ $\qquad$ in terms of the wave potential, $\phi(x, z, t)$.
10) Given velocity vector $\mathrm{V}=(\mathrm{u}, \mathrm{v}, \mathrm{w})$ where $u=2 x y z+7, v=x^{2} z$, and $w=x^{2} y+4$. This flow [is] [is not] irrotational. This flow [does] [does not] satisfies continuity.

## Part B:

## Problem 2: (15\%)

Due to a nearby storm, waves in the central Atlantic Ocean are propagating towards the east coast of the US with frequency $\omega=1.8 \mathrm{rad} / \mathrm{s}$ and amplitude $a=1$ meters. These waves continue towards shore without decaying significantly. The central Atlantic can be assumed to have a deep depth.
a) Does the linear approximation hold for this case? Would it hold for waves with amplitude $a=2$ meters? Justify your answer.
b) Determine the shallowest acceptable depth for the waves to propagate through such that the deep water dispersion relationship still holds?
c) Determine the total average energy per unit area available. Determine the total average kinetic energy per unit area.

## Problem 1: (30\%)

A fluid with density $\rho_{1}$ flows through a circular nozzle as shown in the figure below. The pressure difference between sections 1 and 2 is measured using a manometer filled with a liquid with density $\rho_{2}$. Find the pressure difference $p_{1}-p_{2}$ and the area ratio $A_{1} / A_{2}$ in terms of $\mathrm{Z}_{1}-\mathrm{Z}_{2}, \mathrm{D}, \rho_{1}, \rho_{2}, \mathrm{~V}_{1}$.


Figure by MIT OCW.

## Problem 3: (25\%)

The lock system between the Charles River and Boston harbor is designed to control the flow of water from the river due to an extreme change in the level of the water in Boston harbor between low and high tide. The tides in Boston Harbor change 15 feet each day from low to high tide. The level of the river is a constant 18 feet deep all day long and at low tide the harbor is 14 feet deep.


The lock consists of two swinging gates that are 22 feet wide and extend to the river/harbor seabed. No water can "leak" through the locks when they are closed. The two walls are 50 feet apart as shown in the drawing below:
When a ship wants to go from the harbor to the river, one side of the locks is opened to allow the water in the middle chamber to equalize to the same height as the harbor. When the water in the chamber is at the same level then the ship enters the locks. The gate is closed behind him. A pump then begins to fill or drain the chamber in order to equalize the depth in the chamber to the depth of the river. Once the water level in the chamber is equal to the height of the water in the river, the gate on the river side is opened and the vessel can exit safely. When the vessel has exited the gate closes and the water in the chamber remains at that height until the next vessel goes through.
a) Determine the maximum possible force on the gate adjacent to the harbor when it is closed. Does this occur at high or low tide and at what level is the water in the middle chamber? (Hint: Ignore dynamic forces due to filling the chamber, and or currents in the harbor.)
b) Where is this force acting (center of pressure)?
c) You are asked to redesign the lock gates and are given three options sketched below. Use your intuition and knowledge of hydrostatics to determine which would be the most feasible design. It is possible that there is more than one design that will work.


