21 Hurricane Winds

A 1959 paper by Isaac Van der Hoven gives the spectrum of wind speeds during Hurricane Connie, measured on a tower at Brookhaven National Laboratory. His curve for $S^+(\omega)$ is approximated by the points below:

Frequency, cycles/hr	$S^+(\omega)$ m^2/s
0	0.00
10	0.50
14	0.65
20	1.00
32	2.80
50	3.10
72	2.80
100	2.00
141	1.60
200	1.20
316	0.80
500	0.60
717	0.50
1000	0.40
1410	0.20
2000	0.00

This one-sided spectrum is given in units of m^2/s, i.e., velocity squared divided by ω (rad/s), so that the area under it is equal to the variance. The mean wind speed during most of the hurricane was $13m/s$, but for one hour at the peak it was $20m/s$.

1. Make a plot of this spectrum data - $S^+(\omega)$ vs. ω (rad/s).

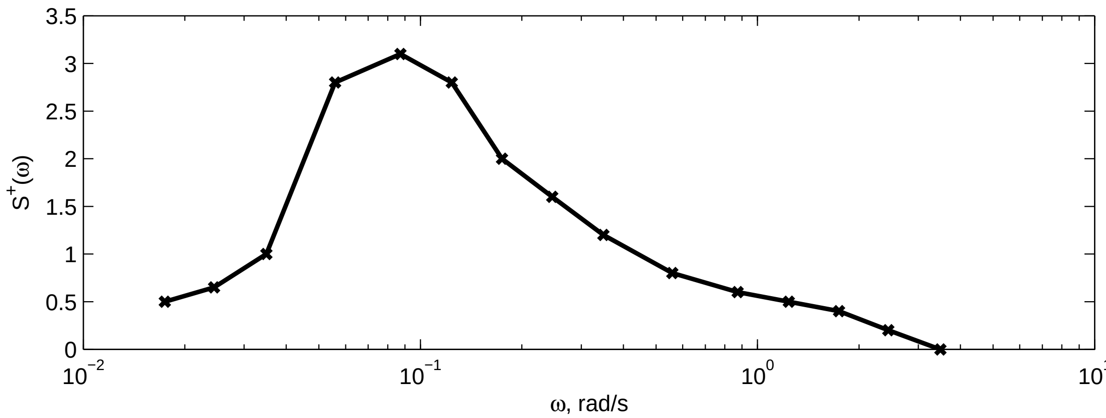
2. What is the characteristic frequency of the windspeed fluctuations? What is the approximate standard deviation of wind velocity, and the significant amplitude $\bar{a}^{1/3}$?

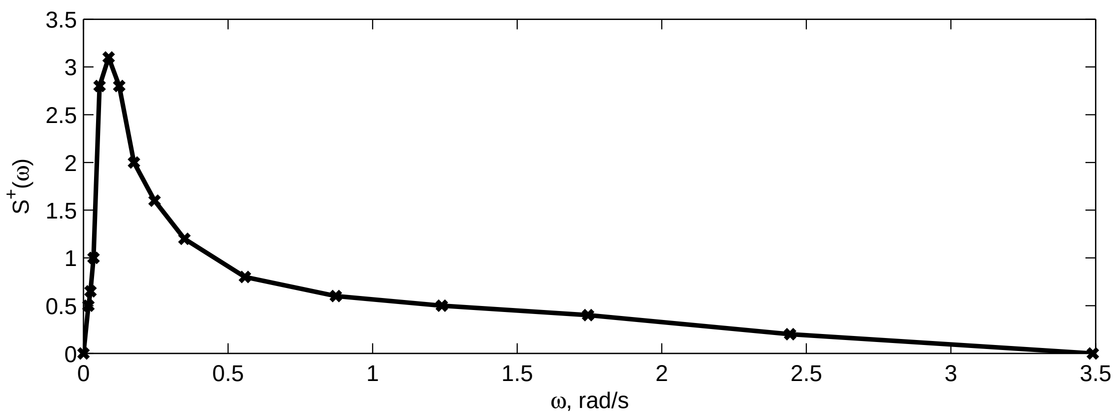
   Solution: The peak frequency is apparently at about fifty cycles per hour, or one cycle per 72 seconds. To get σ and $\bar{a}^{1/3} = 2\sigma$, we have to get the area under the spectrum. The attached code shows how to do this - see also the worked example on the Bretschneider spectrum. The standard deviation here is 1.35m/s, leading to a significant amplitude of 2.7m/s. This is a fluctuation of plus or minus 15-20% from the mean speeds during the hurricane.

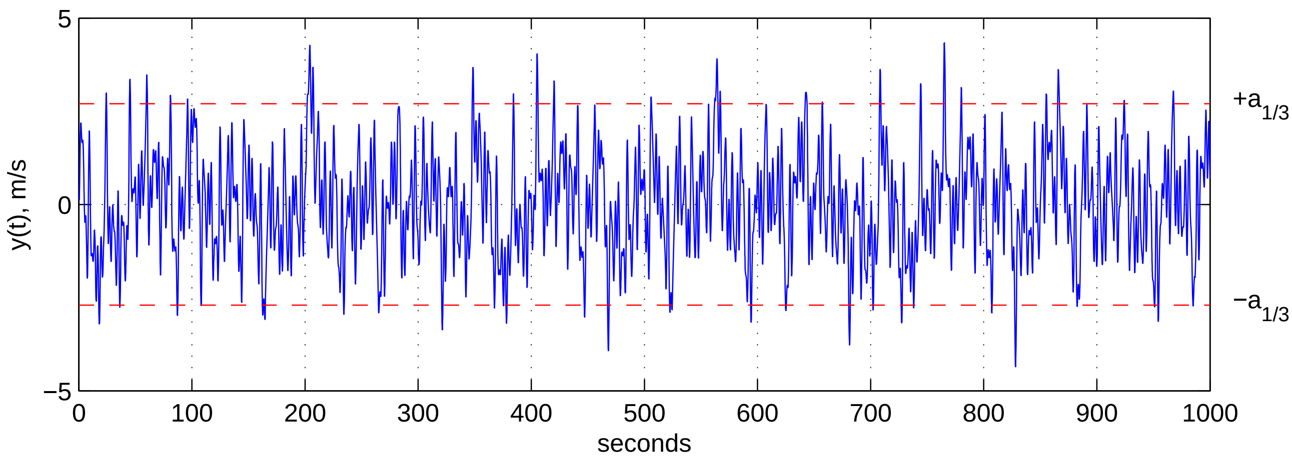
3. Generate a sample trace of time-domain data, with a time step of 0.1 seconds, and a duration of one thousand seconds. Note that for each frequency bin of width $\delta\omega$, we have $a_i^2/2 = S^+(\omega_i)\delta\omega$. This gives you the amplitudes for each center frequency

you use; impose a fixed random phase angle for each component, add the components together, and you are done.

Plot plus and minus $\bar{a}^{1/3}$ on top of your trace, and label.







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% Explore the wind spectrum for Hurricane Connie, after
% van der Hoven 1959.

clear all;

cph = [0 10 14 20 32 50 71 100 141 200 ...
        320 500 710 1000 1400 2000] ; % freq., in cycles per hour
S = [0 .5  .65 1, 2.8 3.1, 2.8  2 ...
    1.6 1.2 .8 .6 .5 .4 .2 0] ; % spectrum to go with cph frequencies

w = cph*2*pi/3600 ; % freq., radians/second

figure(1);clf;hold off;
subplot(212);
plot(w,S,'x-','LineWidth',2) ;
xlabel('\omega, rad/s');
ylabel('S^+(\omega)');
subplot(211);
semilogx(w,S,'x-','LineWidth',2);
xlabel('\omega, rad/s');
ylabel('S^+(\omega)');
print -deps hurricaneWindSpectrum1.eps

widths = ([0 diff(w)] + [diff(w) 0])/2 ; % make the strip widths
var = sum(S.*widths) ; % the variance

stddev = sqrt(var);
asig = 2*stddev ;
disp(sprintf('The stddev is %g m/s and the sig. amp. is %g m/s',...
    stddev,asig));

% compute the amplitudes that go with each frequency, and pick
% some random phase angles, uniformly distributed in [0,2*pi]
for i = 1:length(widths),
    a(i) = sqrt(2*S(i)*widths(i)) ;
    ph(i) = rand*2*pi ;
end;

dt = .1 ; % time step

% a typical two-loop construction to generate the time series
t0 = clock ;
for j = 1:10001, % loop through the times
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    z(j) = 0 ;
    t(j) = (j-1)*dt ; % time
    for i = 1:length(widths), % add up the components
        z(j) = z(j) + a(i)*cos(w(i)*t(j) + ph(i)) ;
    end;
end;
disp(sprintf('The two-loops took %g seconds.', ...
    etime(clock,t0)));

% NOTE:  here is a better way to do the above double loop.  It is
% vectorized and will run much faster (about 100x here) !
t0 = clock ;
j = 1:10001 ;
z = zeros(size(j));
t = dt*(j-1) ;
for i = 1:length(widths),
    z = z + a(i)*cos(w(i)*t + ph(i)) ;
end;
disp(sprintf('The vectorized version took %g seconds.', ...
    etime(clock,t0)));

figure(2);clf;hold off;
subplot(211);
plot(t,z) ;
grid;
hold on;
plot([min(t) max(t)], asig*[1 1],'r--');
plot([min(t) max(t)], -asig*[1 1],'r--');
text(1020,asig,'+a_{1/3}');
text(1020,-asig,'-a_{1/3}');
xlabel('seconds');
ylabel('y(t), m/s');
print -depsc hurricaneWindSpectrum2.eps ;

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