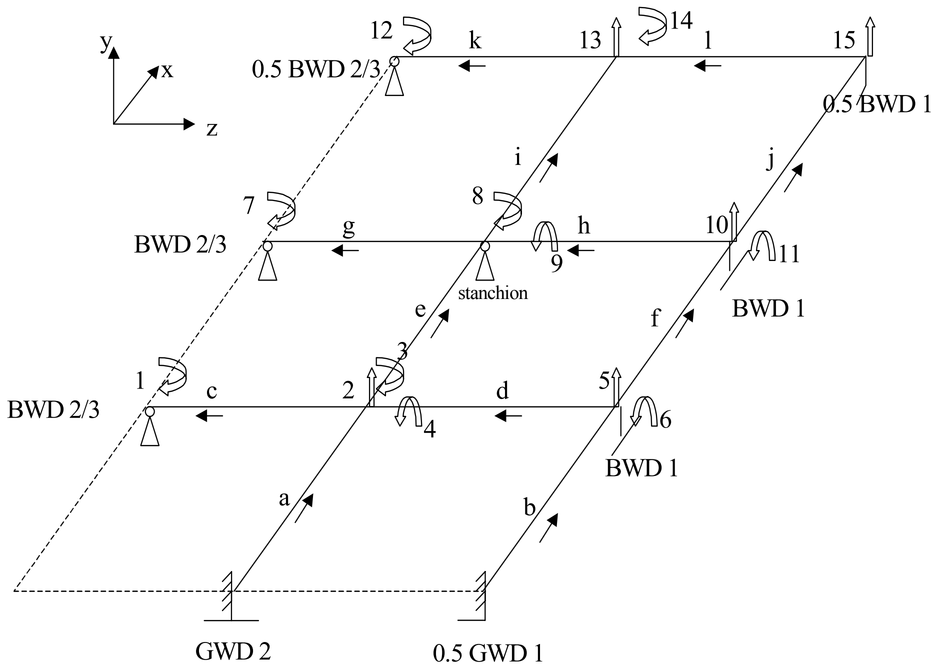
Example of Grillage Analysis

Step 1: Define the Structural Model

weather deck; "pinned" at deck edge, (dof 1,7,12), longitudinal symmetry at 0.5 module length (dof 12,13,15), transverse symmetry at CL (dof 5,6,10,11,15), clamped at transverse bulkhead.

ORIGIN := 1



elements a -> l (1 ->12)
dof 1 -> 15

$n_{elements} := 12$

$n_{nodes} := 15$

Structural properties

Table 1

	L (ft)	I_e (in4)	c_f (in)	M_P (in*lbf)
GWD1	8	455.2	-12.945	
GWD2	8	561.0	-13.123	
BWD1	11.11	437.9	-12.626	$1.818*10^6$
BWD2/3	16.66	215.3	-11.325	$1.058*10^6$

$p_1 = 2.01$ psi; $p_2 = 2.01$ psi; $\sigma_T = 13.782$ ksi; $\sigma_C = -11.489$ ksi; all HTS in mathcad terms:

$\sigma_Y := 47000$ $\sigma_T := 13782$ $\sigma_C := -11489$

#40	#24	#40	#49
$I_{B1} := 437.9$	$I_{B2} := 215.3$	$I_1 := 455.2$	$I_2 := 561.0$
$c_{fb1} := -12.626$	$c_{fb2} := -11.325$	$c_{fg1} := -12.945$	$c_{fg2} := -13.123$

$B_1 := 11.11 \cdot 12$ $B_2 := 16.66 \cdot 12$ $A := 96$

$M_{P1} := 1.818 \cdot 10^6$ $M_{P2} := 1.058 \cdot 10^6$ plastic moment

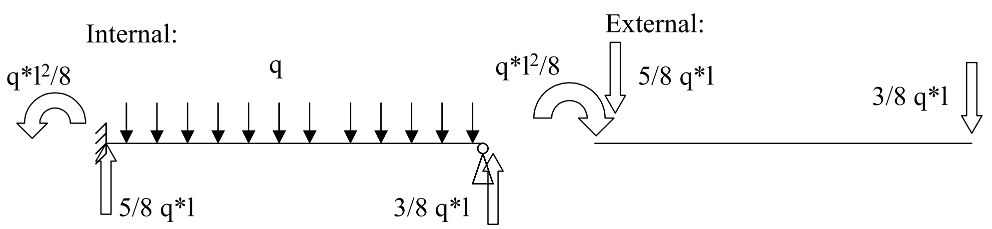
$p_1 := 2.01$ $p_2 := 2.01$

$ie := 1 .. n_{elements}$ $E_{ie} := 29.6 \cdot 10^6$

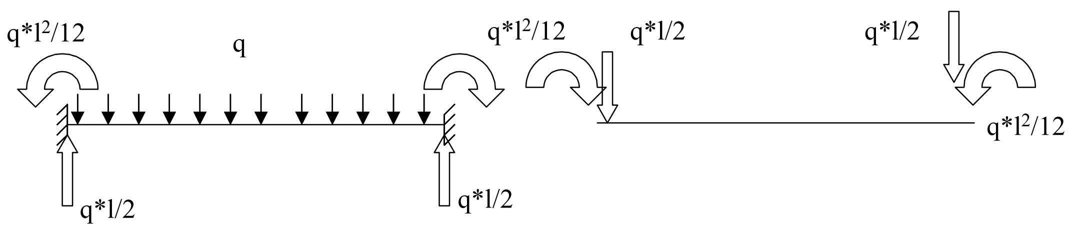
element properties:

$$l := \begin{pmatrix} A \\ A \\ B_2 \\ B_1 \\ A \\ A \\ B_2 \\ B_1 \\ A \\ A \\ B_2 \\ B_1 \end{pmatrix} \qquad I := \begin{pmatrix} I_2 \\ \frac{I_1}{2} \\ I_{B2} \\ I_{B1} \\ I_2 \\ \frac{I_1}{2} \\ I_{B2} \\ I_{B1} \\ I_2 \\ \frac{I_1}{2} \\ \frac{I_{B2}}{2} \\ \frac{I_{B1}}{2} \end{pmatrix} \qquad c_f := \begin{pmatrix} c_{fg2} \\ c_{fg1} \\ c_{fb2} \\ c_{fb1} \\ c_{fg2} \\ c_{fg1} \\ c_{fb2} \\ c_{fb1} \\ c_{fg2} \\ c_{fg1} \\ c_{fb2} \\ c_{fb1} \end{pmatrix}$$

convert distributed load to equivalent static



Clamped pinned



Clamped clamped

MathCad version for internal
moment distribution, q to be defined:

$m := 1 .. n_{elements}$ $q_m := 0$

$$M_{CC}(m,x) := -.5 \cdot q_m \cdot \left(l_m\right)^2 \cdot \left[\frac{1}{6} - \frac{x}{l_m} + \left(\frac{x}{l_m}\right)^2\right]$$

$$M_{CP}(m,x) := \frac{-q_m}{8} \cdot \left[\left(l_m\right)^2 - 5 \cdot x \cdot l_m + 4 \cdot x^2\right]$$

q, and F defined:

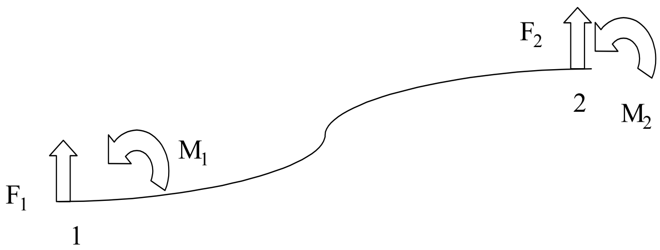
loads and F vector at nodes:
uses static equivalent and weighted average

loads from Faulkner distribution q=pb/2;
"forces" developed from static
equivalence for pinned/clamped beam
and clamped clamped beam:

$$q := \begin{pmatrix} \frac{p_1 \cdot B_1 + p_2 \cdot B_2}{4} \\ \frac{p_1 \cdot B_1}{4} \\ \frac{p_2 \cdot A}{2} \\ \frac{p_1 \cdot A}{2} \\ \frac{p_1 \cdot B_1 + p_2 \cdot B_2}{4} \\ \frac{p_1 \cdot B_1}{4} \\ \frac{p_2 \cdot A}{2} \\ \frac{p_1 \cdot A}{2} \\ \frac{p_1 \cdot B_1 + p_2 \cdot B_2}{4} \\ \frac{p_1 \cdot B_1}{4} \\ \frac{p_2 \cdot A}{4} \\ \frac{p_1 \cdot A}{4} \end{pmatrix} \qquad F := \begin{bmatrix} 0 \\ -.5 \cdot \left(1.25 \cdot q_3 \cdot l_3 + q_1 \cdot l_1 + q_5 \cdot l_5 + q_4 \cdot l_4\right) \\ \frac{q_4 \cdot \left(l_4\right)^2}{12} - \frac{q_3 \cdot \left(l_3\right)^2}{8} \\ \frac{q_1 \cdot \left(l_1\right)^2 - q_5 \cdot \left(l_5\right)^2}{12} \\ -.5 \cdot \left(q_4 \cdot l_4 + q_2 \cdot l_2 + q_6 \cdot l_6\right) \\ \frac{q_2 \cdot \left(l_2\right)^2 - q_6 \cdot \left(l_6\right)^2}{12} \\ 0 \\ \frac{q_8 \cdot \left(l_8\right)^2}{12} - \frac{q_7 \cdot \left(l_7\right)^2}{8} \\ \frac{q_5 \cdot \left(l_5\right)^2 - q_9 \cdot \left(l_9\right)^2}{12} \\ -.5 \cdot \left(q_8 \cdot l_8 + l_{10} + q_6 \cdot l_6\right) \\ \frac{q_6 \cdot \left(l_6\right)^2 - q_{10} \cdot \left(l_{10}\right)^2}{12} \\ 0 \\ -.5 \cdot \left(1.25 \cdot q_{11} \cdot l_{11} + q_9 \cdot l_9 + q_{12} \cdot l_{12}\right) \\ \frac{q_{12} \cdot \left(l_{12}\right)^2}{12} - \frac{q_{11} \cdot \left(l_{11}\right)^2}{8} \\ -.5 \cdot \left(q_{10} \cdot l_{10} + q_{12} \cdot l_{12}\right) \end{bmatrix}$$

Step 2: Determine Element Stiffness Matrix in Structural Coordinates

doing element and structure in one step



Flexure-only beam element

(5.3.1) flexure-only element stiffness matrix

$$kel_{ie} := \frac{E_{ie} \cdot I_{ie}}{\left(l_{ie}\right)^3} \cdot \begin{bmatrix} 12 & 6 \cdot l_{ie} & -12 & 6 \cdot l_{ie} \\ 6 \cdot l_{ie} & 4 \cdot \left(l_{ie}\right)^2 & -6 \cdot l_{ie} & 2 \cdot \left(l_{ie}\right)^2 \\ -12 & -6 \cdot l_{ie} & 12 & -6 \cdot l_{ie} \\ 6 \cdot l_{ie} & 2 \cdot \left(l_{ie}\right)^2 & -6 \cdot l_{ie} & 4 \cdot \left(l_{ie}\right)^2 \end{bmatrix}$$

flex only beam

$n_{nodes\_pe} := 4$

define topology matrix for each element from structure

using n_{nodes} in place of zero to avoid 0 subscript on K. After assembly delete row and column n_{nodes}

structure:

$n_{nodes} = 15$ $\quad n_{nodes} := n_{nodes} + 1$ $\quad n_{nodes} = 16$

increment to allow for "zero" equivalent

from observing the structure.
N.B. elements need to be oriented such that element dof matches structure dof.

$$Top := \begin{pmatrix} n_{nodes} & n_{nodes} & 2 & 4 \\ n_{nodes} & n_{nodes} & 5 & 6 \\ 2 & 3 & n_{nodes} & 1 \\ 5 & n_{nodes} & 2 & 3 \\ 2 & 4 & n_{nodes} & 9 \\ 5 & 6 & 10 & 11 \\ n_{nodes} & 8 & n_{nodes} & 7 \\ 10 & n_{nodes} & n_{nodes} & 8 \\ n_{nodes} & 9 & 13 & n_{nodes} \\ 10 & 11 & 15 & n_{nodes} \\ 13 & 14 & n_{nodes} & 12 \\ 15 & n_{nodes} & 13 & 14 \end{pmatrix}$$

dof for element

Due to the simplicity of the element equations the assembly procedure is relatively straight forward.

First, zero the global stiffness matrix: $\quad K_{n_{nodes}, n_{nodes}} := 0$

Next, assemble the global stiffness matrix element by element:

$n_{el} := n_{elements}$ $\quad n_{el} = 12$ $\quad ie := 1 .. n_{el}$

$i := 1 .. n_{nodes\_pe}$ $\quad j := 1 .. n_{nodes\_pe}$

$$K_{Top_{ie,i}, Top_{ie,j}} := K_{Top_{ie,i}, Top_{ie,j}} + \left(kel_{ie}\right)_{i,j}$$

remove "zero" i.e. n_{nodes} row and column:

reset n_{nodes} $\quad n_{nodes} := n_{nodes} - 1$ $\quad n_{nodes} = 15$

$$K := \text{submatrix}\left(K, 1, n_{nodes}, 1, n_{nodes}\right)$$

Step 4: Apply the Support Conditions

$K \cdot \Delta = F$

Step 5: Solve for the displacements and if desired the reaction forces

$\Delta := K^{-1} \cdot F$

$$\Delta = \begin{pmatrix} 4.647 \times 10^{-4} \\ -0.082 \\ 3.014 \times 10^{-4} \\ 6.781 \times 10^{-5} \\ -0.184 \\ -2.385 \times 10^{-3} \\ -1.204 \times 10^{-3} \\ 2.408 \times 10^{-3} \\ -2.713 \times 10^{-4} \\ -0.344 \\ -1.215 \times 10^{-3} \\ -2.07 \times 10^{-4} \\ -0.115 \\ 2.133 \times 10^{-3} \\ -0.416 \end{pmatrix}$$

Step 6: Obtain element displacement and consequent internal forces (or stresses)

$j := 1 .. n_{elements} \qquad i := 1 .. n_{nodes\_pe}$

this is a reversing of the topology from structure to element:
+1 from 0 elimination approach above

$$\delta_{i,j} := \text{if}\left(Top_{j,i} < n_{nodes} + 1, \Delta_{Top_{j,i}}, 0\right)$$

flexure-only matrix coefficient in relationship: v"(x) =B*δ (5.3.10)

$$B_e(m,x) := \begin{bmatrix} -\dfrac{6}{(l_m)^2} + \dfrac{12 \cdot x}{(l_m)^3} \\ -\dfrac{4}{l_m} + \dfrac{6 \cdot x}{(l_m)^2} \\ \dfrac{6}{(l_m)^2} - \dfrac{12 \cdot x}{(l_m)^3} \\ -\dfrac{2}{l_m} + \dfrac{6 \cdot x}{(l_m)^2} \end{bmatrix}$$

M(x) =E*I*v"(x) for external problem

$$M_E(m,x) := E_m \cdot I_m \cdot B_e(m,x) \cdot \delta^{\langle m \rangle}$$

Step 7: If equivalent nodal loads were used to represent distributed loads, superimpose the internal forces.

clamped clamped elements:

$$M_{CC}(m,x) := -.5 \cdot q_m \cdot (l_m)^2 \cdot \left[\frac{1}{6} - \frac{x}{l_m} + \left(\frac{x}{l_m}\right)^2\right]$$

adding external and internal "forces"

$$M\_tot\_cc(m,x) := M_E(m,x) + M_{CC}(m,x)$$

clamped pinned elements:

$$M_{CP}(m,x) := \frac{-q_m}{8} \cdot \left[(l_m)^2 - 5 \cdot x \cdot l_m + 4 \cdot x^2\right]$$

$$M\_tot\_cp(m,x) := M_E(m,x) + M_{CP}(m,x)$$

now apply structural evaluation criteria to worst case within element (beam); GYTF, GYCF, FYTF, FYTCF & FCPH:
we'll show one example of each of girder (all clamped clamped) and frame - clamped clamped and clamped pinned:

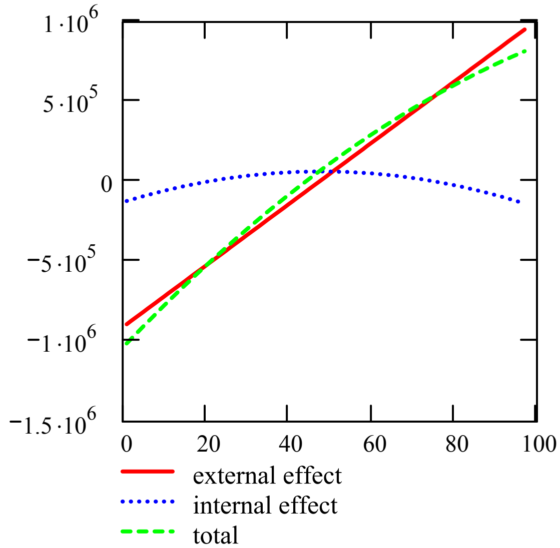
GWD2: element a (n=1):

element a is clamped clamped girder

$n := 1 \qquad x := 1, 2 .. l_n + 1$

tension in flange: GYTF; get maximum tensile stress from maximum positive moment - accounts for c_f

$MM_x := M\_tot\_cc(n, x-1) \qquad \sigma_{bfT_n} := \frac{-\max(MM) \cdot c_{f_n}}{I_n}$



bending

total axial with primary

$\sigma_{bfT_n} = 1.883 \times 10^4 \qquad \sigma_{xfT} := \max\left(\begin{pmatrix} \sigma_T + \sigma_{bfT_n} \\ 0 \end{pmatrix}\right)$

$\sigma_{xfT} = 3.261 \times 10^4 \qquad \gamma R_{GYTF_n} := \frac{\sigma_{xfT}}{\sigma_Y} \cdot 1.25 \qquad \gamma R_{GYTF_n} = 0.867$

compression in flange:

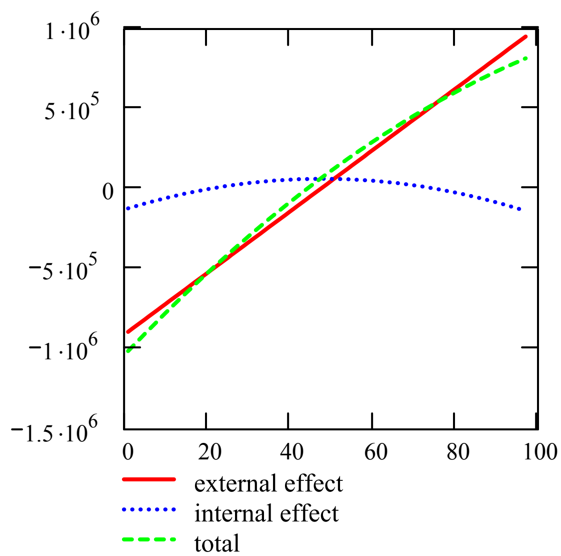
$\sigma_{bfC_n} := \frac{-\min(MM) \cdot c_{f_n}}{I_n} \qquad \sigma_{bfC_n} = -2.43 \times 10^4$

$\sigma_{xfC} := \min\left(\begin{pmatrix} \sigma_{bfC_n} + \sigma_C \\ 0 \end{pmatrix}\right) \qquad \sigma_{xfC} = -3.579 \times 10^4 \qquad \gamma R_{GYCF_n} := \frac{\sigma_{xfC}}{-\sigma_Y} \cdot 1.25 \qquad \gamma R_{GYCF_n} = 0.952$

BWD1: element d clamped clamped frame:

$n := 4 \qquad x := 1, 2 .. l_n + 1$

$MM_x := M\_tot\_cc(n, x-1) \qquad \sigma_{bfT_n} := \frac{-\max(MM) \cdot c_{f_n}}{I_n}$



tension:

$\sigma_{bfT_n} = 7.215 \times 10^3 \qquad \sigma_{xfT} := \max\left(\begin{pmatrix} \sigma_{bfT_n} \\ 0 \end{pmatrix}\right)$

$\sigma_{xfT} = 7.215 \times 10^3 \qquad \gamma R_{FYTF_n} := \frac{\sigma_{xfT}}{\sigma_Y} \cdot 1.25 \qquad \gamma R_{FYTF_n} = 0.192$

compression:

$\sigma_{bfC_n} := \frac{-\min(MM) \cdot c_{f_n}}{I_n} \qquad \sigma_{bfC_n} = -1.349 \times 10^4 \qquad \sigma_{xfC} := \sigma_{bfC_n}$

$\sigma_{xfC} := \min\left(\begin{pmatrix} \sigma_{bfC_n} \\ 0 \end{pmatrix}\right) \qquad \sigma_{xfC} = -1.349 \times 10^4 \qquad \gamma R_{FYCF_n} := \frac{\sigma_{xfC}}{-\sigma_Y} \cdot 1.25 \qquad \gamma R_{FYCF_n} = 0.359$

plastic hinge:

$MMM_x := |MM_x|$ $\gamma R_{FCPH_n} := \frac{\max(MMM)}{M_{P1}} \cdot 1.5$ $\gamma R_{FCPH_n} = 0.386$

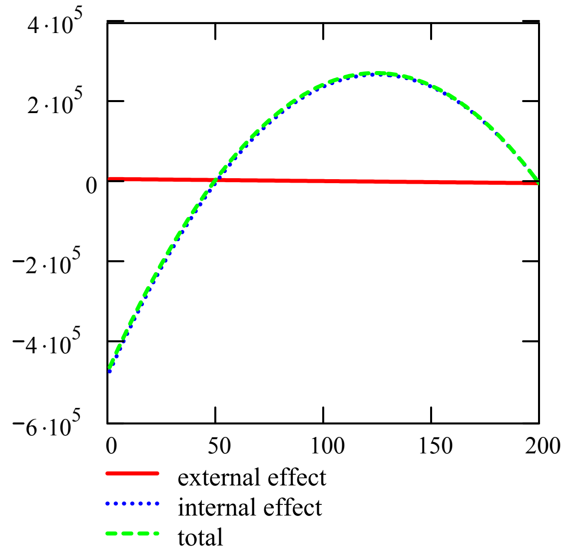
$MMM_x := |MM_x|$ $\gamma R_{FCPH_n} := \frac{\max(MMM)}{M_{P1}} \cdot 1.5$ $\gamma R_{FCPH_n} = 0.386$

BWD2: element c, clamped pinned frame:

$M\_tot\_cp(m,x) := M_E(m,x) + M_{CP}(m,x)$

$n := 3$ $x := 1, 2 .. l_n + 1$

$MM_x := M\_tot\_cp(n, x-1)$ $T_n := \frac{-\max(MM) \cdot c_{f_n}}{I_n}$



tension:

$\sigma_{bfT_n} = 1.447 \times 10^4$ $\sigma_{xfT} := \max\left(\begin{pmatrix} \sigma_{bfT_n} \\ 0 \end{pmatrix}\right)$

$\sigma_{xfT} = 1.447 \times 10^4$ $\gamma R_{FYTF_n} := \frac{\sigma_{xfT}}{\sigma_Y} \cdot 1.25$

compression

$\sigma_{bfC_n} := \frac{-\min(MM) \cdot c_{f_n}}{I_n}$ $\sigma_{bfC_n} = -2.481 \times 10^4$ $\sigma_{xfC} := \sigma_{bfC_n}$ $\gamma R_{FYTF_n} = 0.385$

$\sigma_{xfC} := \min\left(\begin{pmatrix} \sigma_{bfC_n} \\ 0 \end{pmatrix}\right)$ $\sigma_{xfC} = -2.481 \times 10^4$ $\gamma R_{FYCF_n} := \frac{\sigma_{xfC}}{-\sigma_Y} \cdot 1.25$ $\gamma R_{FYCF_n} = 0.66$

plastic hinge:

$MMM_x := |MM_x|$ $\gamma R_{FCPH_n} := \frac{\max(MMM)}{M_{P2}} \cdot 1.5$ $\gamma R_{FCPH_n} = 0.669$