2.092/2.093

FINITE ELEMENT OF SOLIDS AND FLUIDS I

FALL 2009

Homework 6- solution

Instructor: Prof. K. J. Bathe Assigned: Session 14
TA: Seounghyun Ham Due: Session 16

Problem 1 (20 points):

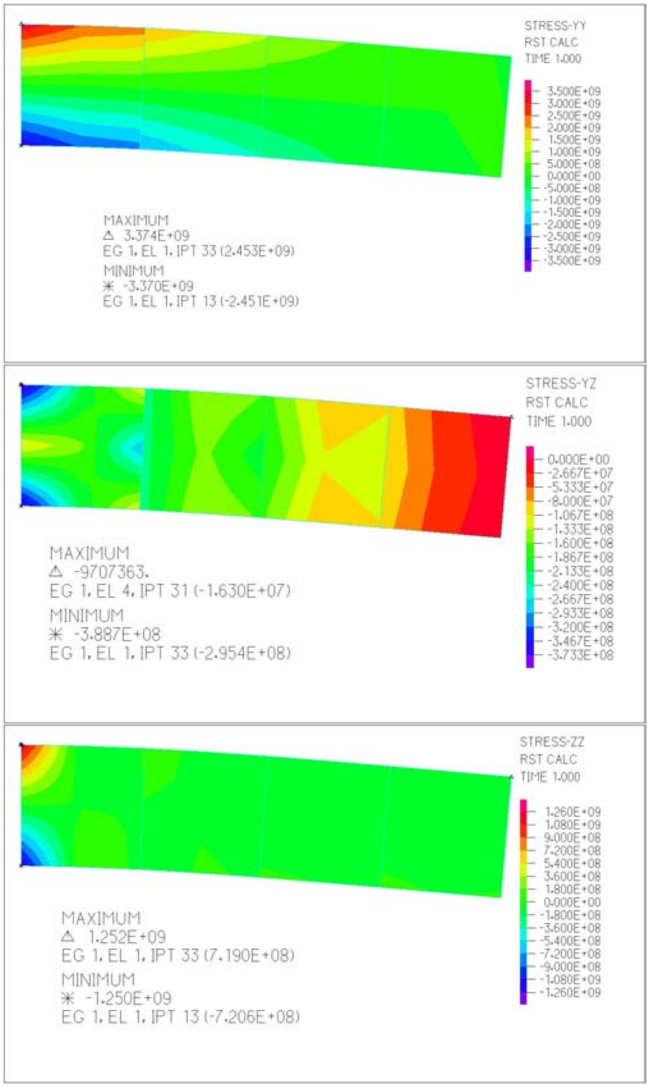

Figure 1: Stress distributions with 1×4 mesh

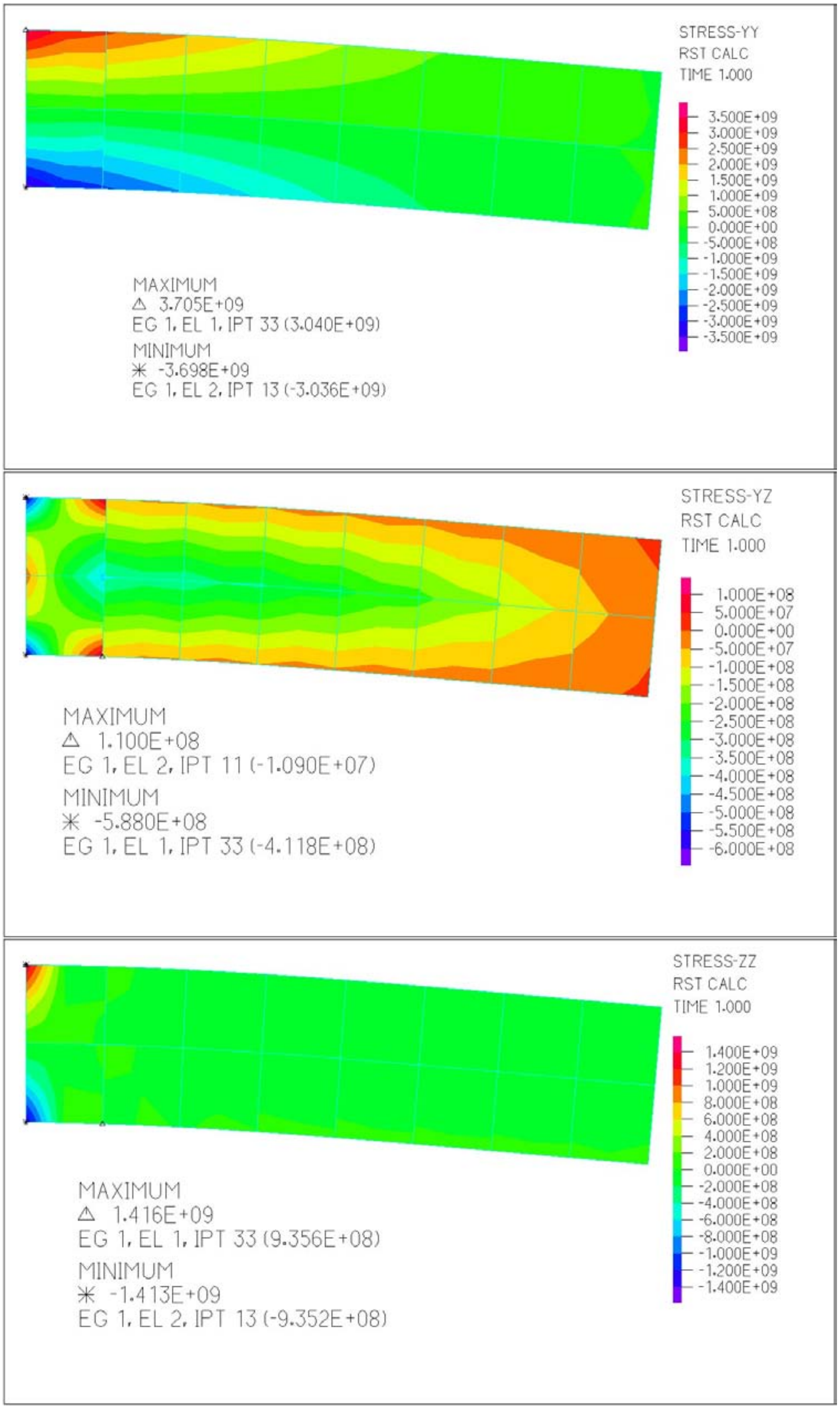

Figure 2: Stress distributions with 2×8 mesh

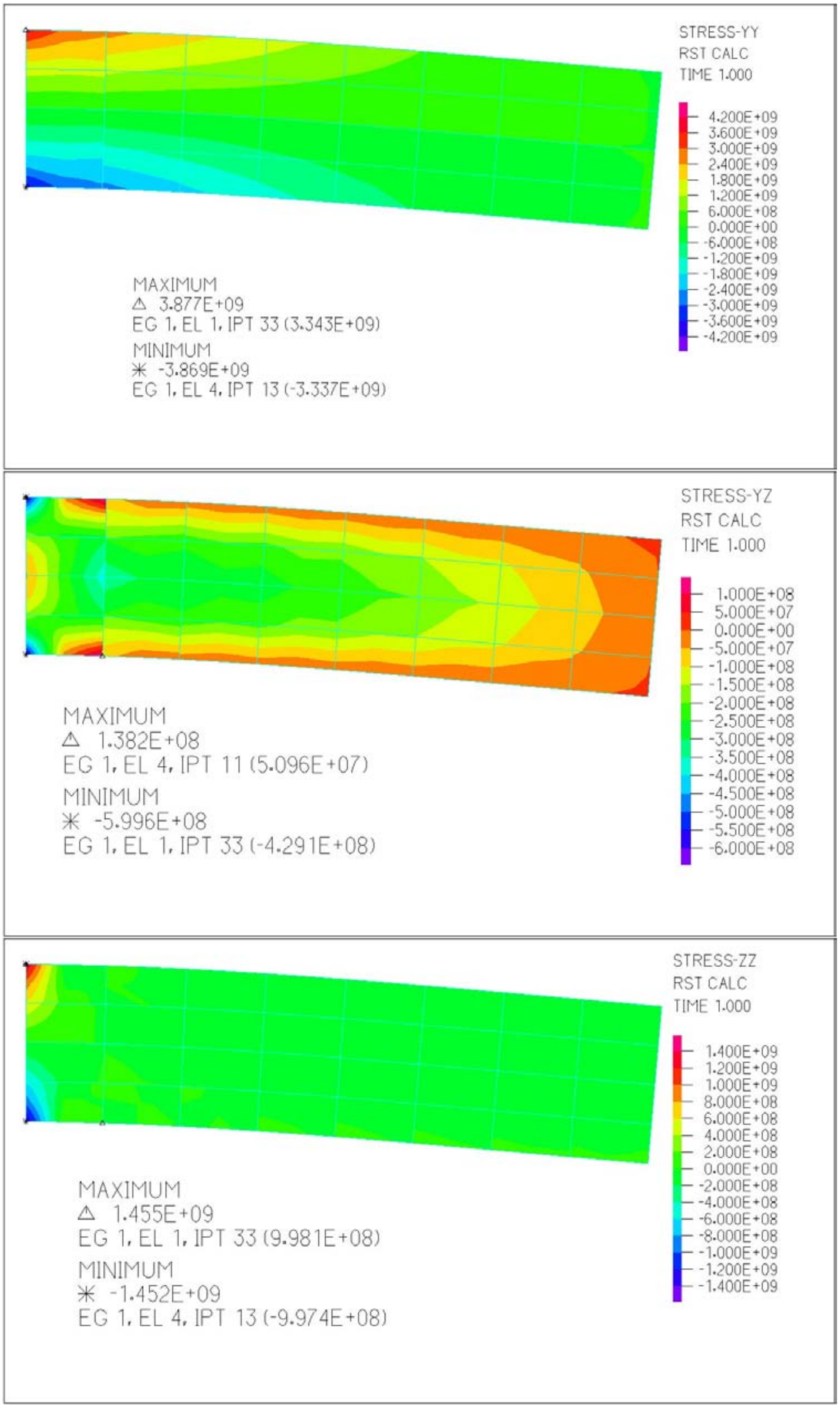

Figure 3: Stress distributions with 4×8 mesh

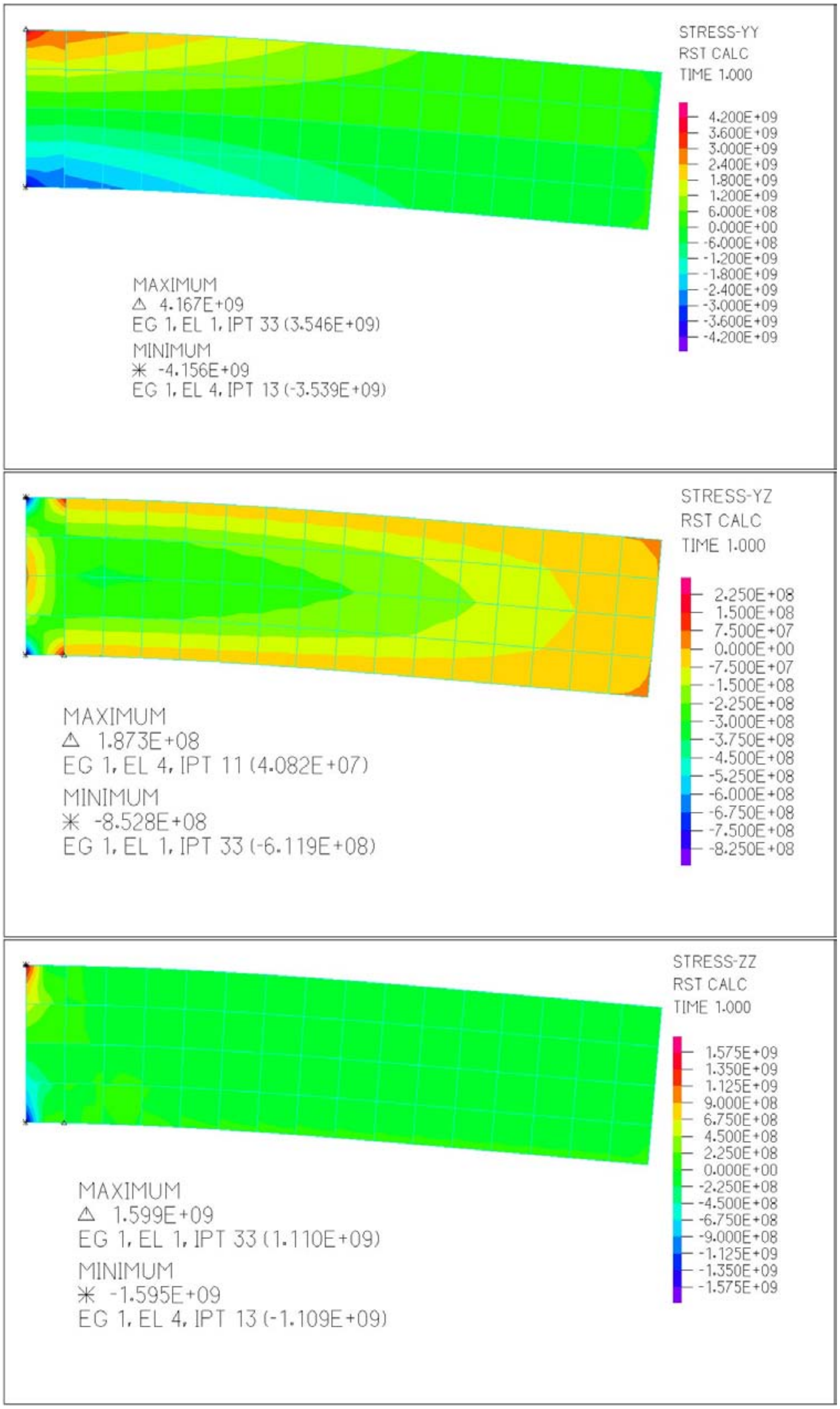

Figure 4: Stress distributions with 4×16 mesh

· Euler-Bernoulli Beam Theory

Governing equation: $\mathrm{EI}\dfrac{\mathrm{d}^4\mathrm{w}}{\mathrm{dx}^4}=\mathrm{p(x)}=-\dfrac{\mathrm{p}_0}{\mathrm{L}}\mathrm{x}$.

Boundary conditions: $w(0)=0$; $\dfrac{dw(0)}{dx}=0$; $\dfrac{d^2w(L)}{dx^2}=0$; $\dfrac{d^3w(L)}{dx^3}=0$.

After solving the governing equation with boundary conditions, we are able to obtain

$\tau_{xx}=-\mathrm{Ez}\dfrac{\mathrm{d}^2\mathrm{w}}{\mathrm{dx}^2}=\mathrm{Ez}\left(\dfrac{\mathrm{p}_0}{2\mathrm{EIL}}\right)\left(\dfrac{\mathrm{x}^3}{3}-\mathrm{L}^2\mathrm{x}+\dfrac{2}{3}\mathrm{L}^3\right)$ and $\tau_{xx}=\dfrac{2\mathrm{p}_0\mathrm{L}^2}{\mathrm{h}^2}=3.2\times10^9\ \mathrm{N}/\mathrm{m}^2$ at x=0,

$z=h/2$.

Table 1: Numerical results for maximum stresses

Mesh	Max. stress τ_{yy}	Max. stress τ_{yz}	Max. stress τ_{zz}
1×4	3.37E+09	9.71E+06	1.25E+09
2×8	3.71E+09	1.10E+08	1.42E+09
4×8	3.88E+09	1.38E+08	1.46E+09
4×16	4.17E+09	1.87E+08	1.60E+09

The finite analysis stress, τ_{yy}, appears to keep increasing to infinite stress at the top corner of the beam as the number of elements increases. This is because of the stress concentration caused by the geometry discontinuity. However, in real the stress never reach infinity since the material will yield before this point.

Problem 2 (20 points):

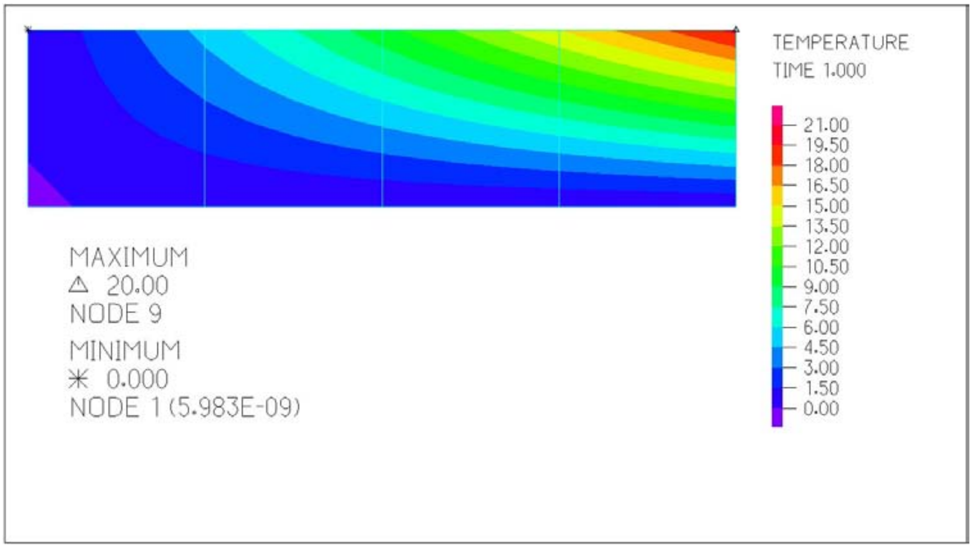

Figure 5: Temperature distribution with 1×4 mesh

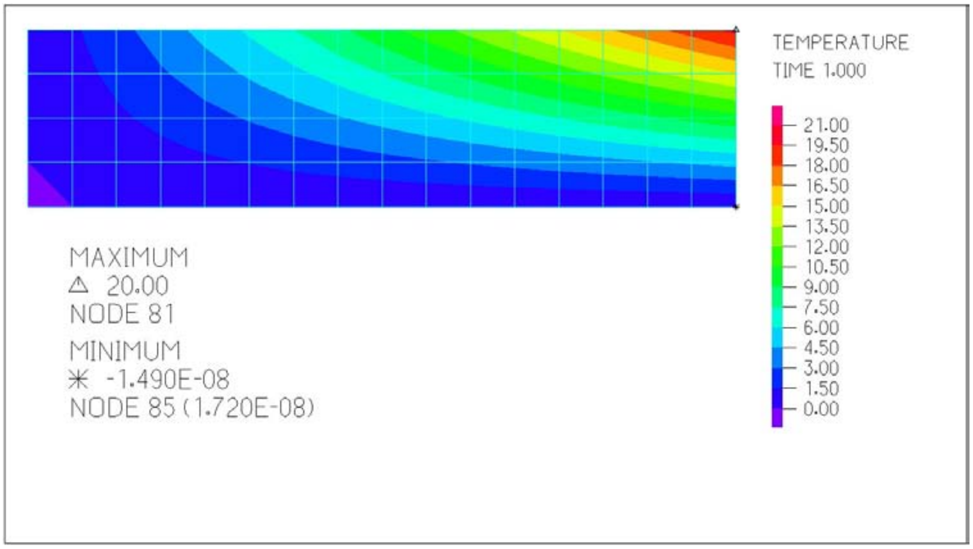

Figure 6: Temperature distribution with 4×16 mesh

The temperature distribution obtained from finite analysis seems to have converged even with a very coarse mesh.

Problem 3 (20 points):

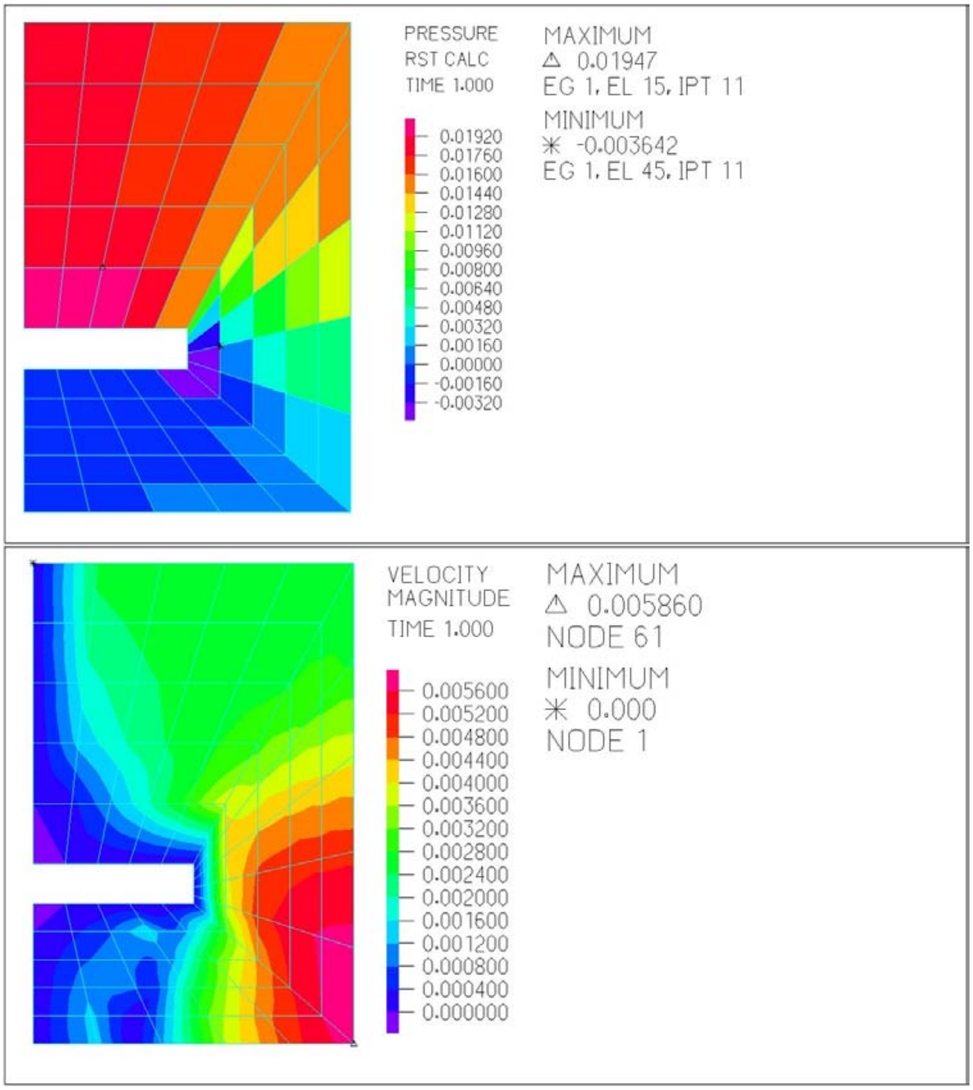

Figure 7: Velocity and pressure distributions with 75 4-node elements

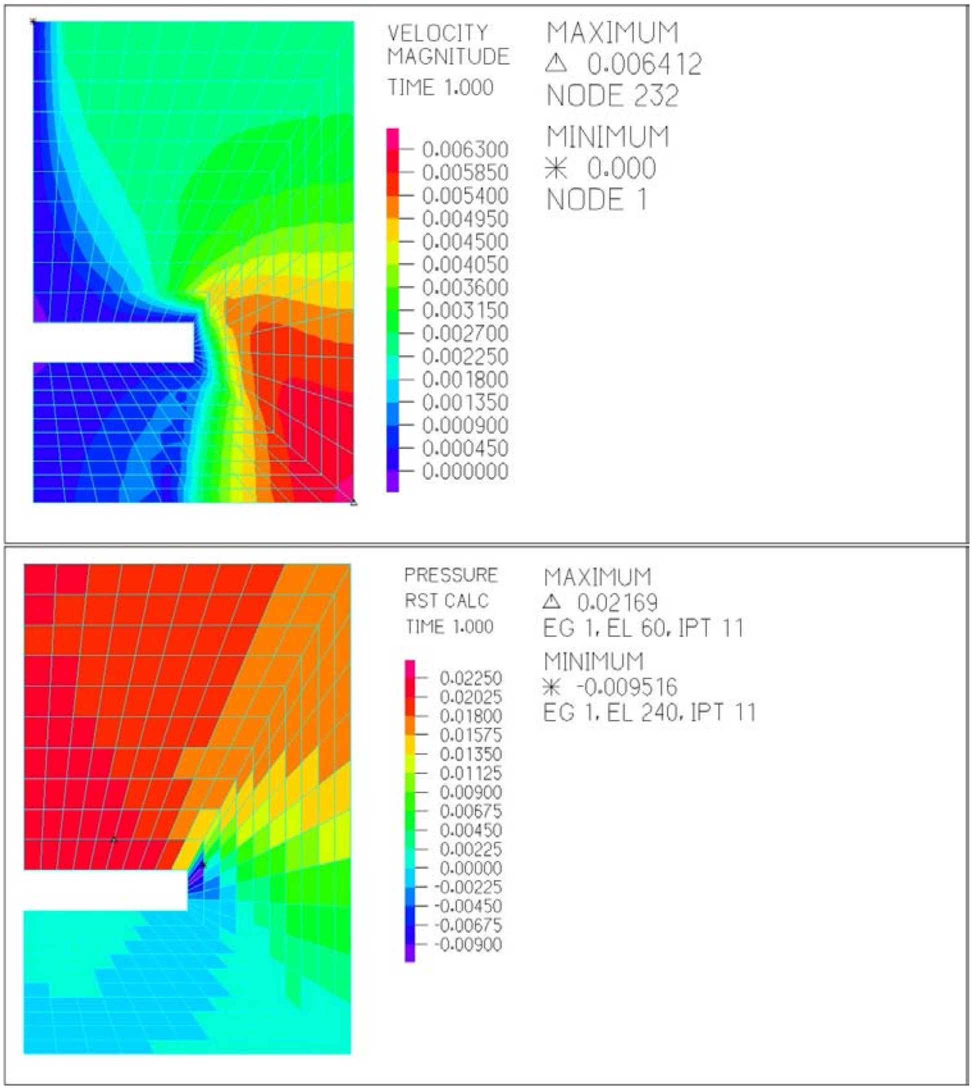

Figure 8: Velocity and pressure distributions with 300 4-node elements

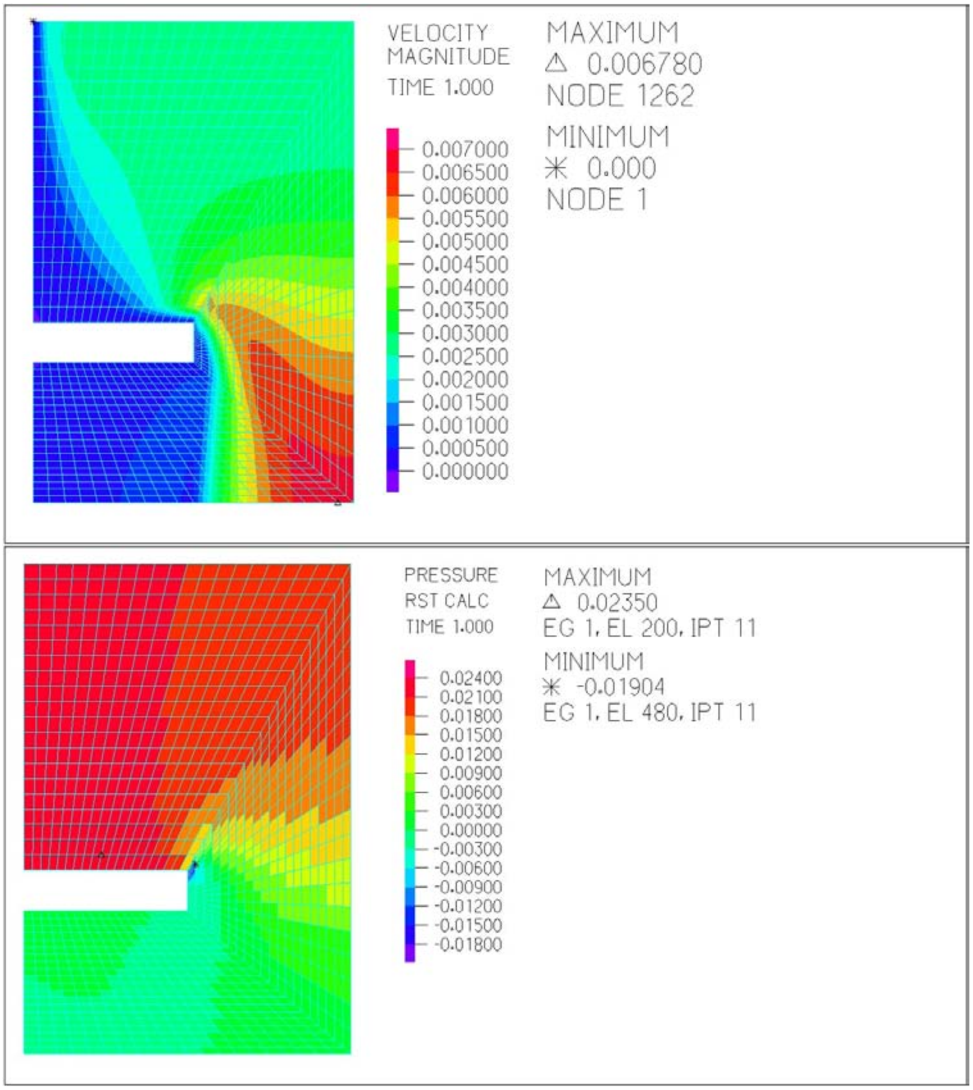

Figure 9: Velocity and pressure distributions with 1200 4-node elements

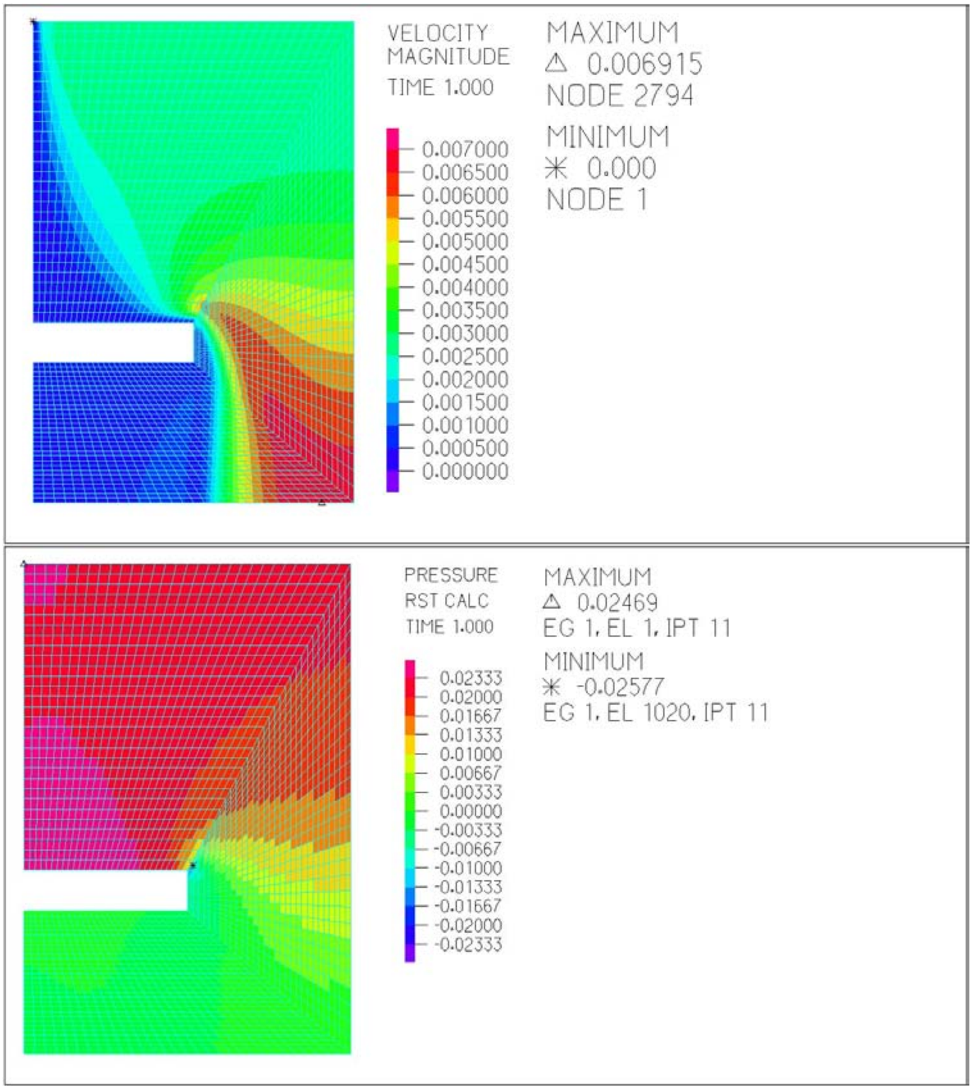

Figure 10: Velocity and pressure distributions with 2700 4-node elements

The velocity and pressure distributions are converging as the number of elements increases. Since the Reynolds number featuring this flow is relatively low, the laminar flow assumption is reasonable.

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